THE GRAPHIC METHOD

BY

INFLUENCE LINES

FOR

BRIDGE AND ROOF COMPUTATIONS

BY

WILLIAM H. BURR, C.E.

Professor of Civil Engineering in Columbia University in the City of New York
Member of the American Society of Civil Engineers; Member of the Institution of Civil Engineers of Great Britain

AND

MYRON S. FALK, PH.D.

Lecturer in Civil Engineering in Columbia University in the City of New York
Associate Member of the American Society of Civil Engineers

SECOND EDITION

FIRST THOUSAND

NEW YORK

JOHN WILEY & SONS

LONDON: CHAPMAN & HALL, LIMITED

1908

ROBERT DRUMMOND, PRINTER, NEW YORK

PREFACE.

This book exhibits an entirely modern graphical treatment, by the method of influence lines, of simple statically determinate structures such as bridges and roof-trusses, three-hinged arches, cantilevers, and other constructions of the same general class. The simplicity, elegance, and generality of procedures by influence lines and areas afford quick and eminently practical methods of computing stresses in all forms of trusses, whether with straight, curved, or broken outlines and with any single or multiple systems of bracing, whether square or skew, or with any manner of loading whatever. The ease and clearness with which the greatest stresses, both main and counter, in web and chord members, may be determined for any specified system of loading has already brought the treatment into general use in practical structural design. Taken as a whole the method commends itself not only as a simple universal system of analysis, but also as a remarkably labor-saving means of making computations in structural work. Some knowledge of elementary algebraic methods, including the method of moments or sections, is required, but nothing more.

The graphical method of finding the deflections of all points of trusses and other structures, given in Chapter V, has also marked advantages of extreme simplicity, generality, and saving of labor, when compared with the alge-

braic method. The effect of the strain in every member of the structure has its proper place and value in the final result, thus securing complete accuracy.

The reader of the book will find that the methods of treatment set forth in it cover plate girders and solid beams, as well as articulated or jointed structures like ordinary bridge- and roof-trusses. The stresses in roof-trusses caused by the flexure of the supporting columns are treated in detail.

Sufficient problems are introduced to illustrate clearly the practical applications of the various methods of treatment, but it should always be borne in mind that the greatest advantage will accrue to those students who work out original problems and make independent practical applications.

The same general observations hold in connection with the complete design of the members of a railroad bridge given at the end of the book, to which, in the determination of the stresses, the previously deduced methods are applied.

The application of these lines to statically indeterminate structures, such as two-hinged and fixed ended arches, swing bridges, suspension bridges, and certain types of cantilevers, will appear in another volume.

W. H. B.
M. S. F.

Columbia University, February 2, 1905.

PREFACE TO SECOND EDITION.

In issuing the second edition of this book, use is made of the opportunity to correct such typographical errors as inevitably are found in the diagrams and numerical work necessarily employed in the preparation of such a work. All such corrections have been made so far as they have been discovered, and it is believed that no material inaccuracies of that character remain.

The method of determining deflections of trusses has been extended to include those with subdivided panels. It is believed that as this type of truss is much used at present the new material will be found of practical value.

The largest addition of new matter consists of the eighteen pages devoted to suspended cantilevers. With the extension of bridge work into the field of long spans, the treatment of this type of structure adds materially to the scope and value of the book.

W. H. B.
M. S. F.

Columbia University, January 8, 1908.

CONTENTS.

CHAPTER I.

GENERAL PRINCIPLES OF GRAPHIC STATICS, AND APPLICATION TO ROOF-TRUSSES.

CHAPTER II.

INFLUENCE LINES FOR SIMPLY SUPPORTED BRIDGE-TRUSSES.

CHAPTER III.

THE THREE-HINGED ARCH.

CHAPTER IV.

CANTILEVERS.

CHAPTER V.

DEFORMATION OF TRUSSES.

CHAPTER VI.

THE DETAILED DESIGN OF A RAILROAD BRIDGE.

GRAPHIC METHODS FOR BRIDGE AND ROOF COMPUTATIONS.

CHAPTER I.

GENERAL PRINCIPLES OF GRAPHIC STATICS, AND APPLICATION TO ROOF-TRUSSES.

Graphic statics is an adaptation of graphical methods to the study of statics, so that the values of quantities otherwise found algebraically may be determined by the aid of geometrical diagrams or figures.

Art. 1.—Definitions.

A force is completely known when its magnitude, direction, and point of application are given, but the line of action without the point of application is generally sufficient. So far as any problems in statics are concerned, a force may be considered as acting at any point along its line of action. The location of the point of application of a force affects only the material upon which the force acts.

The magnitude, direction, and line of action of a force may be represented by a straight line, the length of the line representing the magnitude by a proper scale. The pointing of the line with an arrow-head, or a proper reading of the letters placed at the ends of the line, will indicate

the direction of the force. In Fig. 1 the force P acting in the direction of the arrow must be read *ab* and not *ba*.

Forces may be coplanar or non-coplanar, and either concurrent or non-concurrent. Coplanar forces have lines of action all in one plane, while non-coplanar forces may have lines of action anywhere in space; for the present the latter will not be considered. Concurrent forces have lines of action all meeting in a point, but the lines of action of non-concurrent forces do not meet in a single point.

The resultant of any system of forces is a single force

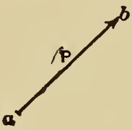


FIG. 1.

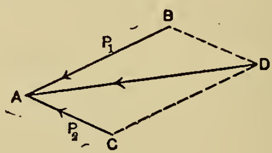


FIG. 2.

which, having the same effect as to equilibrium or motion as the system itself, may replace it.

In accordance with the principles of composition and resolution of forces, if P_1 and P_2 (Fig. 2) are two concurrent forces acting at A in the directions BA and CA respectively, and if P_2 be transposed so that CA becomes the side DB of the triangle ABD, then will DA represent the resultant in magnitude and direction with its point of application at A.

It is seen that in following around the sides of the triangle, the resultant must be taken in a direction opposite to that of its component forces. The forces AB and AC are said to be the components of the force AD, and by drawing other triangles upon AD as a base an indefinite number of pairs of other components may be found. Components are called rectangular when they are at right angles with each other. It should be carefully noted that the direction of a force is always given by the proper

reading of the letters placed at the ends of the line representing it.

Art. 2.—Resultant of any Number of Forces.

The method of finding the resultant of two concurrent forces is easily adapted to finding the resultant of any number of concurrent forces. Let the forces P_1, P_2, $P_3 \ldots P_4$ (Fig. 3) act at A, and let them be transferred to Fig. 4, so as to form the sides of the polygon $BCD \ldots$, the forces being directed around the polygon in the same way. It is seen that R_1 is the resultant of P_1 and P_2, its direction in the triangle BCD being from B to D. Simi-

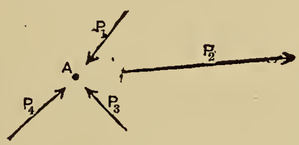


FIG. 3.

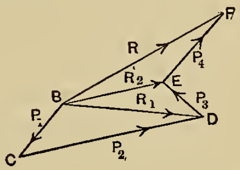


FIG. 4.

larly R_2 is the resultant of R_1 and P_3, and finally R or BF is the resultant of all the given forces, and it acts in a direction opposite to that of the other sides of the polygon.

In order to find the resultant of any number of concurrent forces, it is necessary only to lay down the forces as the sides of an open polygon, their directions following around the polygon in the same sense. The closing side of the polygon will then represent the resultant in magnitude and its direction must be opposite to that of the other forces. The polygon may have re-entrant angles, or the sides may cross, but the form of the figure does not affect the result. The order in which the forces are laid down is also a matter of indifference, but to avoid confusion they are usually taken consecutively about any point.

A force may be held in equilibrium by another force, equal to it in magnitude and having the same line of action, but opposite in direction. Therefore if the direction of the resultant found in Fig. 4 be reversed, the system of forces will be held in equilibrium by FB, since FB exactly balances BF. Any system of concurrent forces, therefore, is in equilibrium if the separate forces may be laid down as the sides of a closed polygon and if the lines of action of these forces follow around the polygon in the same way. Such a figure is called a force or equilibrium polygon, and it represents graphically two of the three equations of condition for equilibrium of coplanar forces, viz., first, that the sum of the horizontal components of the forces must equal zero, and secondly, that the sum of the vertical components must equal zero.

Art. 3.—Application of Force Polygon to Finding of Unknown Quantities.

Since the force polygon represents two equations of condition, the finding of two unknown quantities in a system of concurrent forces in equilibrium is made possible. Therefore if all the forces in such a system are known except two, and if the lines of action of these two be given, the unknown quantities may be found by means of the force polygon. Let P_4 and P_5 (Fig. 5) represent the lines of action of these two unknown forces, the forces P_1, P_2, and P_3 being fully known. If the force polygon (Fig. 6) for P_1, P_2, and P_3 be drawn and if the figure be closed by drawing lines parallel to P_4 and P_5 at the open ends of the polygon, P_4 and P_5 will be completely determined. It is obvious that two solutions leading to the same results are possible, one of which is shown by the broken lines. In using the force polygon it will be found convenient to take the forces in the order of their rotation as they act

about the point of application, although that is evidently not necessary.

In a similar manner the force polygon furnishes a solution in the case of a system of concurrent forces, if all the forces except one are completely known. In that case the closing side of the polygon fully determines the resultant in amount and direction.

It is evident that the two equations of condition for equilibrium represented by the method of the force polygon

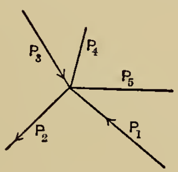


FIG. 5.

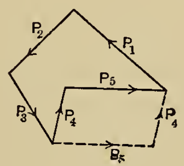


FIG. 6.

must also be applicable to non-concurrent forces; but a third equation of condition must also hold, viz., the sum of the moments of all the forces taken about any point in their plane must be equal to zero. The graphical treatment of this condition will be considered later.

Art. 4.—System of Notation.

The ease of operating with graphical methods is due in a great measure to the system of notation invented by Bow, and known by his name. Two diagrams are employed, one showing only the lines of action of the forces in connection with the structure, and called the truss diagram, while the other is simply the force polygon already explained.

The space between the lines of action of any two adjacent forces on the truss diagram, or the space on that diagram within any polygon whose sides are formed by the

lines of action of any forces, is marked by a capital letter. Each force will then always lie between two and only two such capital letters, and consequently it can be designated conveniently by the two letters between which it lies. These letters, changed from capital to small letters, are then placed on the force polygon at the ends of the lines representing the same forces. The correct disposition of the letters at the ends of the lines indicates the direction of action of each force, as has already been explained.

Example.

Fig. 7, representing a simple king-post truss carrying a concentrated load at its centre, illustrates the truss diagram. The external forces acting upon the truss are the two reactions at its ends and the concentrated centre loading. The space between the concentrated load and the left reaction is marked by the letter A, the space above the truss and between the reactions by B, and the space between the concentration and the right reaction by C. The apices of the triangles are numbered 1, 2, 3, and 4. According to this system of notation the left reaction will be known as ab or ba, the load as ac or ca, and the right reaction as cb or bc. The indetermination due to the use of two names for one force is made to vanish by the use of the following rule: In any truss diagram all forces must be read in the same order, clockwise or counter-clockwise, around the figure or about a point.

In Fig. 7, employing the direction indicated by the circular arrow placed in the upper left-hand corner, the names of the three external forces are then immediately fixed, as ab, bc, and ca.

The internal forces, or stresses in the members of the truss, are designated in exactly the same manner, the triangular spaces in the truss being marked D and E. The forces acting about the point 1 are then bd, da, and ab;

those about point 2, *ed*, *db*, and *be*; etc. It is seen that the same force has a different name when read with respect to two different points; thus the member between panel points 1 and 2 is the member or force *bd* with respect to point 1, and *db* with respect to point 2. As will be shown presently, this feature of the method determines at once the character of the stresses in the various members.

Art. 5.—Determination of Kind of Stresses.

Since the structure is in equilibrium under the loading assumed, every panel point is also in equilibrium, and consequently all the forces at any one such point are in equilibrium. A force polygon may therefore be drawn for the concurrent forces acting at each panel point.

The reactions in Fig. 7 may each be found by analytical methods to be equal to one half the centre concentration.

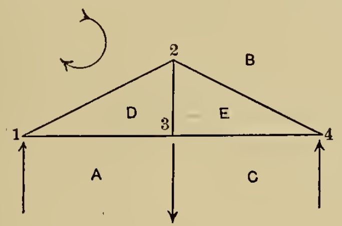


FIG. 7.

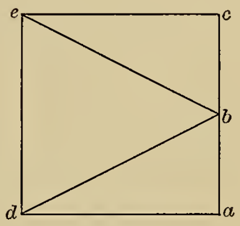


FIG. 8.

There remain, then, at point 1 only two concurrent forces with known lines of action to be determined. The force *ab*, since it acts upwards, is laid down on the force diagram (Fig. 8), with the letter *a* at the bottom of the line representing its magnitude and direction. The line *bd* is then drawn parallel to *BD*, beginning at *b*, and the line *da* parallel to *DA*, beginning at *a*; their intersection will fix the point *d*. Since for equilibrium the forces in this force polygon must follow each other in the same sense, and

since *ab* is fixed, the direction of the force *bd* is from *b* to *d*; transferring this direction to the truss diagram, it will be seen that the force *bd* acts towards the point 1.

It is clear that a force acting towards a point, in the manner of *bd*, represents compressive stress in the member in which it acts, in the same manner that a force acting away from a point represents tensile stress. In the present case *bd* therefore acts in compression with a magnitude represented by the length of the line *bd*. Similarly, transferring the direction of *da* to the truss diagram, it will be found that *da* acts away from 1 and represents a tensile stress. Thus all the forces acting at the point 1 have been fully determined.

Other panel points at which there are but two unknown forces must next be considered in succession.

At panel point 2 the forces must be read *db*, *be*, and *ed*, *bd* being already drawn on the force diagram. A line parallel to *BE* is therefore drawn through *b* and one parallel to *ED* through *d*; then the intersection of these lines furnishes *e* on the force diagram. By transferring the direction of *be* to Fig. 7, it will be found acting towards panel point 2, causing compression. Similarly *ed* acts downwards from point 2 and causes tension.

Panel points 3 and 4 are to be treated similarly, but it is unnecessary to explain the operations in detail.

Art. 6.—Crane-trusses.

The graphic principles thus far explained may be illustrated by an application to a form of truss which has been used for powerful cranes under circumstances requiring much headroom, and shown in Fig. 9. The truss revolves about a vertical axis situated in the centre of the member *BD*. In that example the weight hanging from the peak is supposed to be 20,000 pounds. Each

chord of the truss NA, LA, JA, etc., or MC, KC, IC, etc., is made up of chords of quadrants of two circumferences of circles, the radius for the inner chord being 23.5 feet, and that for the outer chord 26 feet; the member BD is 5 feet.

The lengths of the various members are as follows:

$NA = 5$ feet;
$LA = JA = MC = 6$ feet;
$HA = KC = 7$ feet;
$FA = IC = 8$ feet;
$GC = 9$ feet.

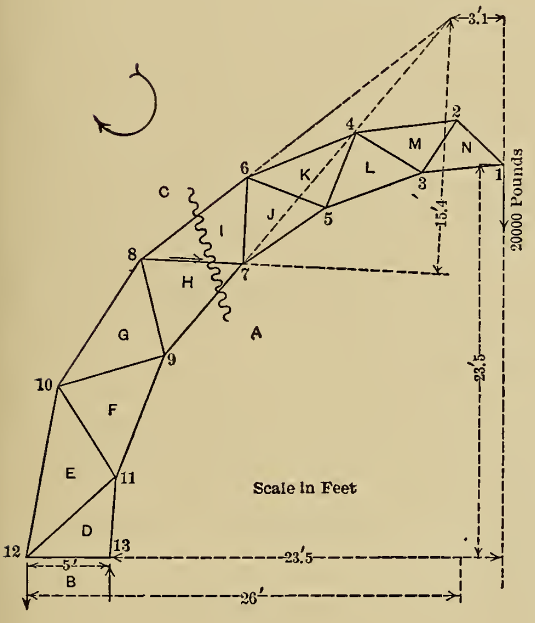


FIG. 9.

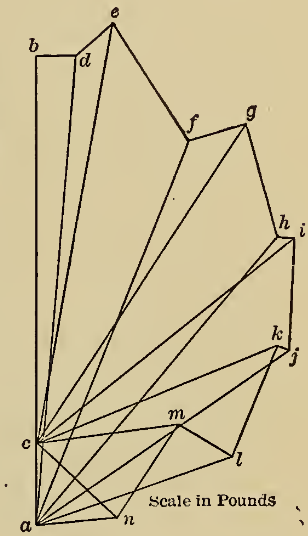


FIG. 10.

Fig. 10 is a complete diagram for the stresses in the truss, supposing the only load to be the 20,000 pounds hanging from the peak. If it should be necessary to take

into account the weight of the truss itself, it would be done by considering the weights of the various members as concentrated at the different panel points, and the problem would be treated precisely as in the case of roof-trusses, to be treated in succeeding articles.

The construction of Fig. 10 is begun by considering the equilibrium of panel point 1; the line *ca* is drawn downward parallel to the line of action of the weight *CA*, and then at its ends are drawn lines parallel to *AN* and *NC*. The order in which forces are taken about any panel point is indicated by the direction of the arrow shown in the upper left-hand corner of Fig. 9. Panel point 1 is the first panel point at which the stresses can be determined, since at that point there are but two unknown quantities. Other panel points are then to be taken in such order that at the point considered there are only two unknown forces. This is imperative. The stresses at panel point 2 are therefore treated, the lines *nm* and *mc* being drawn respectively parallel to the corresponding members in the truss diagram. The other panel points may then be treated in exactly the same manner, considering them in the order in which they are numbered.

In order to determine the character of the stress in any member, such as *MN*, it is only necessary to observe that the direction of *mn* in the force diagram is downward and toward panel point 3. It is in connection with panel point 3 only that the member *MN* is named in that way, and the latter is therefore in compression. If this member were to be named in regard to panel point 2, it would be termed *NM*. Its direction would then be upward with regard to panel point 2, indicating compression as already found.

The diagram gives the following results, the positive sign indicating tensile stress, and the negative sign compressive:

STRESSES IN THE CRANE OF FIGURE 9.

Upper Chord.	Lower Chord.	Web Members.
$CN = +26,000$ lbs.	$AN = -19,000$ lbs.	$NM = -27,000$ lbs.
$CM = +34,000$ lbs.	$AL = -50,000$ lbs.	$ML = +15,000$ lbs.
$CK = +62,000$ lbs.	$AJ = -73,000$ lbs.	$LK = -29,000$ lbs.
$CI = +79,000$ lbs.	$AH = -90,000$ lbs.	$KJ = +\ 3,000$ lbs.
$CG = +91,000$ lbs.	$AF = -99,000$ lbs.	$JI = -27,000$ lbs.
$CE = +102,000$ lbs.	$AD = -112,000$ lbs.	$IH = -\ 4,000$ lbs.
		$HG = -28,000$ lbs.
Reactions.		$GF = -15,000$ lbs.
$BC = \downarrow 94,000$ lbs.		$FE = -34,000$ lbs.
		$ED = -12,000$ lbs.
$AB = \uparrow 104,000$ lbs.		$BD = -\ 9,000$ lbs.

These results can easily be checked by the algebraic method of moments. For instance, the stress in the member IH is found by taking a section through the truss as shown (Fig. 9) and taking moments about the intersection of CI and HA; the lever-arm of IH as scaled from the diagram is 15.4 feet. The only external force to the right of the section is the weight at the peak, which, by scale, has a lever-arm about the centre of moments of 3.1 feet. The equation of moments is then as follows, the direction of IH being indicated by the arrow lying along it:

$$-IH \times 15.4 = 20,000 \times 3.1;$$
$$\therefore \quad IH = -4000 \text{ pounds.}$$

This value checks that obtained by the graphical method.

The last force found graphically is the reaction BC, and if its graphical value be checked by the method of moments, it may be assumed that all previous graphical results are correct. For this member the centre of moments is at panel point 13; the equation of moments is

$$BC \times 5 = 20,000 \times 23.5;$$
$$\therefore \quad BC = 94,000 \text{ pounds.}$$

This result agrees with the value obtained from the diagram.

If a chain, rope, or cable pass along either chord, the tension in it will tend to produce an equal amount of compression in the panels of that chord; the resultant stress, therefore, in any panel will be the algebraic sum of this amount of compression and the stress due to the weight at the peak.

Fig. 11 is a skeleton diagram of an ordinary crane, which revolves about the centre line of 2-3 as an axis. *AB* is the weight hung at the peak, 1. The force diagram, Fig. 12, is found by drawing *ab* vertically downwards equal to the weight, then at *a* and *b* drawing *bc* and *ca* respectively parallel to *BC* and *CA* of Fig. 11. *bc* then represents the

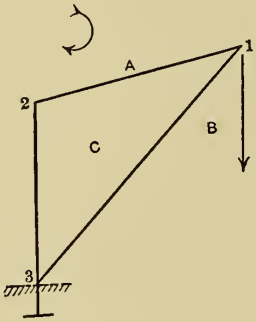


FIG. 11.

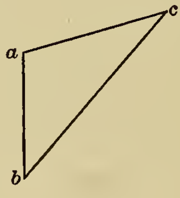


FIG. 12.

compression in *BC*, and *ca* the tension in *CA*. As before, the tension in the rope or cable tends to produce an equal amount of compression in any member along which it lies.

Let l denote the normal distance from the line of action of *CA* to any point in the centre line of the vertical member 2-3; then any section of 2-3 will be subjected to the bending moment

$$M = ca \cdot l,$$

ca representing the stress in *CA*.

2-3 will also be subjected to a direct stress (tension in Fig. 11) equal to the vertical component of the stress in *CA*.

The greatest resultant intensity of stress in any section will be the combination of the intensities due to these two causes.

Art. 7.—Non-concurrent Coplanar Forces, and the Funicular Polygon.

Concurrent forces only have thus far been treated, but the principles employed in the concurrent force diagram are equally applicable to non-concurrent coplanar forces.

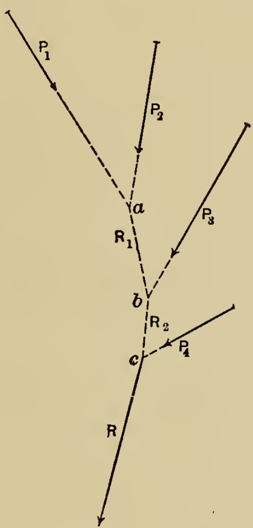


FIG. 13.

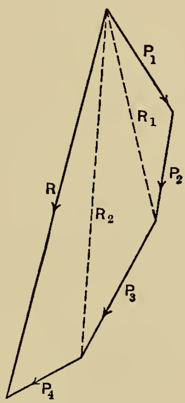


FIG. 14.

In the case of equilibrium of such forces, one other condition must also hold, viz., the sum of the moments of these forces taken about any point as a centre must be equal to zero. This condition may be expressed graphically, as will presently be shown.

Let P_1, P_2, P_3, and P_4 (Fig. 13) represent four non-

concurrent forces whose resultant is to be completely determined. By combining P_1 and P_2 in the force diagram, Fig. 14, their resultant is found to be R_1 and its line of action passes through a at the intersection of P_1 and P_2 in Fig. 13. This resultant may then be combined with P_3 (Fig. 14), producing R_2, whose line of application passes through the point b (Fig. 14), the intersection of R_1 and P_3. Similarly, this resultant may be combined with P_4, producing the final resultant R (Fig. 14), whose line of action passes through the point c (Fig. 13).

This method becomes inapplicable when the forces treated are parallel or nearly parallel, but the following construction, which is perfectly general, may then be employed:

Let Fig. 15 represent five forces P_1, $P_2 \ldots P_5$, whose resultant it is desired to obtain; Fig. 16 shows the value and the direction of its resultant as obtained by the usual force polygon, but both its point of application and line of action (Fig. 15) are still unknown. Let O be any arbitrary point in Fig. 16, and let there be drawn from O to the ends of all the forces $P_1, P_2 \ldots P_5$ radiating lines numbered 1, 2, 3 ... 6. It is then evident that in Fig. 16 each force has been resolved into two components, as P_1 resolved into 1 and 2, and P_2 into 2 and 3, and so on. Choosing any point on P_1, such as a in Fig. 15, let the line 2 be drawn parallel to the line 2 in Fig. 16 until it intersects the line of action of the force P_2; from this intersection point draw 3 parallel to 3 in Fig. 16 until it intersects P_3, and so continue the construction to its completion. Finally produce the lines 1 and 6, Fig. 15, to their intersection at the point b. This is a point in the line of action of the resultant; for at point a in Fig. 15 the force P_1 has been replaced by the force components 1 and 2 acting in the directions indicated by the arrows, these directions being shown in Fig. 16. P_2 has similarly been replaced by the forces 2

and 3 acting in the directions similarly shown; but it is evident that the two forces 2 between P_1 and P_2 balance

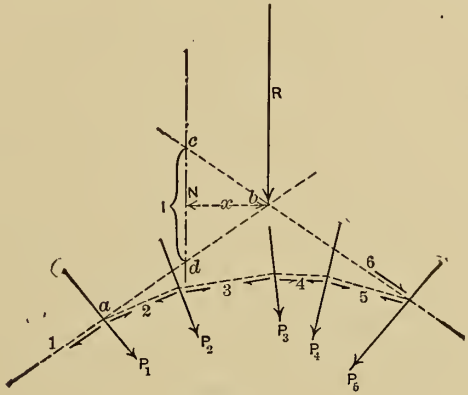


FIG. 15.

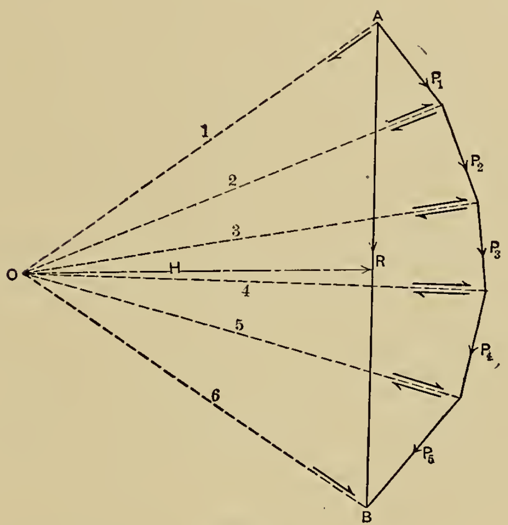


FIG. 16.

each other since they are equal in amount, opposite in direction, and act along the same line. Similarly, the two forces 3 between P_2 and P_3 cancel, as do likewise the pairs

of forces 4 and 5. The five forces P_1, $P_2 \ldots P_5$ have then finally been replaced by the two forces 1 and 6. These, as is evident from Fig. 16, are the components of the final resultant, and may therefore be replaced by the resultant, which must act at their intersection b.

The polygon 1-2-3-4-5-6 drawn in Fig. 15 is termed the polygonal frame or funicular polygon, because it is a frame of members or bars, which, if of sufficient sectional area, will sustain the set of external forces P_1, $P_2 \ldots P_5$ as long as their lines of action and magnitudes remain unchanged. The term funicular polygon is so distinctive that it will hereafter be invariably used for this polygon, although the term polygonal frame is much used. The point O from which the radial lines are drawn is termed the pole, and it is usually designated by the letter O. The distance from O to R is termed the pole distance and is generally represented by H. The radial lines, which are always designated by numbers, are called rays.

These closed diagrams fulfil the condition that the sum of the moments of the forces of the system about any point must equal zero, as will now be shown.

Let it be determined to find the sum of the moments of all the forces P_1, $P_2 \ldots P_5$ about the point N, which is distant x from their resultant R in Fig. 15. Since the moment of the resultant about this point is equal to the sum of the moments of its various components, it will be sufficient to treat only that resultant. The moment is, therefore, $M = R.x$. Draw through the centre of moments N the line cd parallel to the resultant, and continue the lines 1 and 6 until they intercept on cd the distance I. The triangles bcd and OAB (Fig. 16) are similar since their sides are respectively parallel. The following proportion is therefore obtained:

$$x : cd :: H : R,$$

or

$$M = R \, . \, x = cd \, . \, H.$$

Hence the moment of all the forces about the assumed centre, N, is equal to the product of the intercept on the line (Fig. 15) drawn through that centre parallel to the resultant between those rays which are the components of R, multiplied by the pole distance. More simply stated, the moment is the product of an intercept on the truss diagram multiplied by the pole distance of the force diagram.

It is seen that this moment reduces to zero when the centre of moments is taken at the intersection of the two component rays of R; viz., at the point b of Fig 15, or if taken anywhere on the line of action of R, for in either case the intercept equals zero.

If the five forces $P_1, P_2 \ldots P_5$ form a system in equilibrium, the force polygon in Fig. 16 will close and the resultant R will be zero, making the moment $M = R \, . \, x = 0$. Hence the sum of the moments of all the forces of the system about any point N will be zero. Again, in Fig. 15, if the direction of R be reversed, it and the five forces P_1, $P_2 \ldots P_5$ will constitute a system in equilibrium. In that case the funicular polygon becomes a-b-c-5-4-3-2 and it is closed. If a system of forces is in equilibrium, therefore, both the force polygon and the funicular polygon or frame will be closed. These two polygons therefore express graphically the three static equations of condition for equilibrium of non-concurrent forces.

Art. 8.—Some Special Features of the Funicular Polygon.

Although the funicular frame in Fig. 15 was drawn by beginning at a in the line of action of P_1, it is now evident that any point on the line of action of any force might have been taken as a point of beginning, while the pole

and the rays remain unchanged, and that for each such point a different funicular polygon would have been found. With the same pole and the same set of rays, therefore, there may be an infinite number of funicular polygons for the same system of forces in equilibrium; but since the rays are unchanged the corresponding sides of all those polygons will be parallel, or, in other words, those polygons or frames will be concentric.

Again, any point whatever may be taken as the pole O in Fig. 16, and rays may be drawn from any such point to the angles of the force polygon. Hence there are an infinite number of poles for each of which there an an infinite number of concentric funicular polygons for any given system of coplanar forces; but when the position of the pole is changed the corresponding sides of the polygons or frames generally cease to be parallel to each other. The pole, therefore, may be either within or without the force polygon, or at any point of its perimeter.

If the pole is located at an angle of the force polygon, as at B, Fig. 18, the ray drawn to that angle will become zero in length and the corresponding side of the funicular polygon, as 5 in Fig. 17, will disappear. In other words, there will be no stress in the side 5 of Fig. 17, and the directions of P_5 and P_4 will coincide with the directions of sides 1 and 4 respectively, so that the side or bar 5 would be omitted. This case is that of the ordinary masonry arch.

When the pole is changed in position on the force diagram, the pole distance is altered. Since the pole distance appears directly as a factor in determining the moments of any of the forces in the diagram, it follows that with an increase of pole distance there will be a decrease of intercept on the funicular frame; that is, the intercept varies inversely with the pole distance. This consideration is of importance in the treatment of arched ribs, for in those

structures the pole distance represents the horizontal thrust.

If the pole is located in some side of the force polygon, as at some point in P_5 of Fig. 18, two rays will be coincident in direction with that side, and the two sides or bars of the funicular polygon, as 1 and 5 in the case supposed, will coincide in direction with each other and with the force P_5 acting at their common extremity. The funicular polygon will thus apparently have one less side or bar than the number of rays in the force polygon.

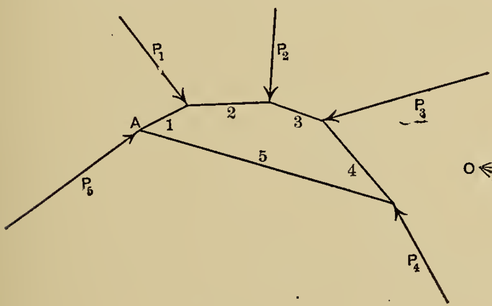


FIG. 17.

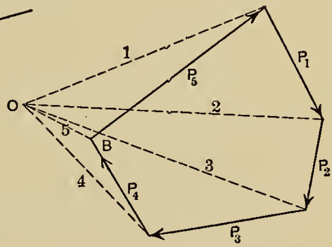


FIG. 18.

Remembering the double significance of the rays as representing the stresses in the bars of the funicular polygon by the same scale as that to which the forces P_1, P_2, etc., are measured, it is well to observe that each side of the force polygon with the two rays adjacent to it form a triangle of forces in equilibrium, which three forces are those meeting at the corresponding angle of the funicular polygon.

In Fig. 17 the external forces are taken as acting inward or towards the frame; but all directions may be reversed and the funicular polygon may be inverted without changing in any respect any of the preceding demonstrations, constructions, or conclusions. If the force directions are reversed and the funicular polygon or frame inverted, the condition shown in Fig. 19 will result, which, if P_5 and P_4

coincide in direction with 1 and 4, so that 5 disappears, is that of the elementary suspension-bridge cable.

It is to be borne in mind that thus far the system of forces supported in equilibrium by the funicular polygon has been supposed to be invariable in every respect. The extreme forces P_5 and P_4 are usually reactions or supporting forces supplied by piers or abutments.

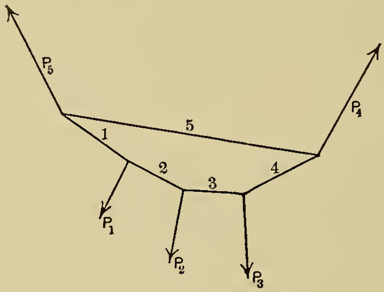


FIG. 19.

Since the force and funicular polygons correspond to three equations of condition of statics, they may therefore be used for finding not more than three unknown quantities, in any problem involving the equilibrium of coplanar forces. Their application to the general solution of some problems of statics may therefore be discussed.

Art. 9.—Lines of Action of all Forces Known, but Magnitudes and Directions of Two Forces Unknown.

In Fig. 20 let the lines of action of all the forces $P_1 \ldots P_5$ be known, but let the magnitudes and directions of any two, as P_4 and P_5, be unknown. Since it is a matter of indifference in what order the forces are taken in the construction of the force polygon, let the known forces be taken in order from P_1 to P_3, as shown. Then the known lines of action of P_4 and P_5 will enable the two remaining sides

of the force polygon to be drawn from A and B, while the directions of those two forces will be determined by traversing the perimeter of the polygon in the way or sense indicated by the known sides, all as shown in Fig. 21. The funicular polygon can then be drawn from any point in any line of action in the manner already established in the previous articles.

Although the unknown forces are shown in Fig. 20 as adjacent to each other, it is clear from the demonstration that they may be any two whatever of the system.

If the unknown forces are parallel to each other, their intersection (that of P_4 and P_5 in the force polygon of

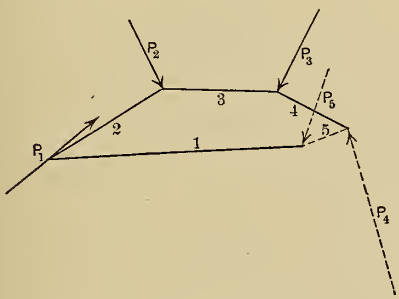


FIG. 20.

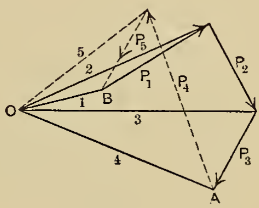


FIG. 21.

Fig. 21) in the force polygon becomes indeterminate. In that case it is only necessary to *complete the funicular polygon* before completing the force polygon, as is illustrated by Fig. 22, in which all the forces are supposed to be parallel to each other. The forces whose lines of action are known but whose magnitudes and directions are unknown are P_4 and P_5. The force polygon now becomes a straight line, since all the forces are parallel, and in order to show completely this coincidence of force lines all the forces in Fig. 23 are represented by close parallel lines. The force (full) lines P_1, P_2, and P_3 are first drawn in any order that will give their proper consecutive directions,

and the rays 1, 2, 3, and 4 are then run from any pole O to the extremities of the former. By commencing at any point in any of the lines of action (all known) the four rays will enable all sides or bars of the funicular polygon, except 5, to be drawn in the usual way. Both extremities A and B of the side 5 will thus be fixed and the line itself can at once be completed. The ray 5 can then be drawn from the pole O to the point C in the straight line which represents the force polygon. The forces P_4 and P_5 will be represented in magnitude by lines extending from C to the extremities of P_3 and P_1, and their directions (P_4 down

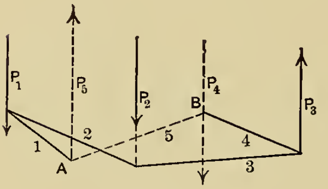


FIG. 22.

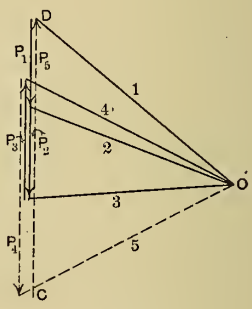


FIG. 23.

and P_5 up) will be found by traversing these lines in the same sense as the known force line $P_1 \ldots P_3$. Both forces and frame are thus completely determined.

It is of the greatest importance to observe that the force polygon for a system of parallel forces reduces to a straight line, which is really composed wholly or in part of two or more coincident straight lines. It is also to be noted that any pole O has been chosen, but that any other of the infinite number of possible poles would lead to precisely the same final results. This latter case of the parallel forces is a common one in practice, but usually the two extreme forces are unknown in magnitude only.

It may be observed that if the directions of the two component rays of either unknown force in the force polygon of Fig. 23 and the magnitude of one of those components are known, the intersection of the line of action of the other component with the force line *CD* will fix the point *C* and determine the unknown forces without the aid of the funicular polygon.

Art. 10.—Moments in a Beam.

The funicular polygon in the case of a beam carrying a system of parallel loads represents the bending moment at every section of the beam. Let Fig. 24 represent a simple non-continuous beam carrying the loads $P_1, P_2 \ldots P_5$ at the points shown. This system of weights is held in equilibrium by the two reactions R and R_1 whose directions and points of application are known, but whose amounts are to be determined by the funicular polygon.

Lay off in regular order the forces $P_1, P_2 \ldots P_5$ in the force diagram Fig. 25, and choose any point O as a pole. Draw the rays 1, 2 . . . 6 and transfer them in the proper manner to Fig. 24. This funicular polygon can only be drawn for the bars 1, 2, 3, 4, 5, and 6, for this disposes of all the known rays; but since the polygon must close, the side 7 is drawn as the closing line and transferred to Fig. 25. Its intersection with the line of loads at m determines the amounts of the reactions R and R_1 as shown.

The moment at any section of the beam, such as xy, may then be obtained by the rule of the "product of intercept by pole distance" (Art. 7). The resultant of the forces R, P_1, P_2, and P_3 to the left of the section is included between the two rays 4 and 7 of Fig. 25; consequently the intercept required lies between the bars 4 and 7 and is the line ab in Fig. 24. The moment at xy is therefore $ab \, . \, H$, where H represents the pole distance (not shown). For parallel forces the pole distance is a constant quantity, and is

generally chosen as a unit force, frequently 1000 or 10,000 pounds, in order to avoid arithmetical computation. In general, then, the moment at the section xy is expressed by the ordinate directly below it, if the significance of the pole

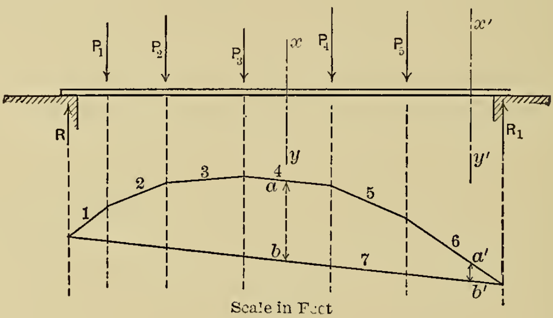


FIG. 24.

distance be remembered. The moment at any other section, such as $x'y'$, is then also expressed by the ordinate below it, or $a'b'$.

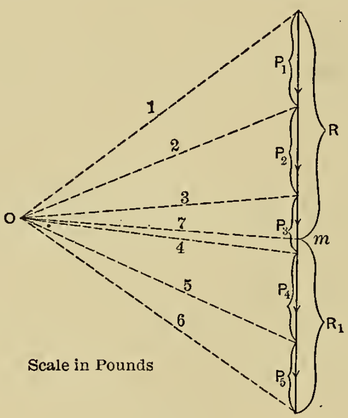


FIG. 25.

The equivalence of the funicular polygon as a moment polygon applies as well to cantilever or continuous beams as to non-continuous beams, provided that the loading consists of a series of parallel weights.

Art. 11.—All Forces Given Except Two, of which the Line of Action of One and the Point of Application of the Other are Known.

This case is represented by Fig. 26, in which $P_1 \ldots P_5$ constitute any system of forces all except two of which, as P_4 and P_5, are known. The line of action of one unknown force, P_5, is given together with the point of application A of the other. That portion of the force polygon representing the known forces $P_1 \ldots P_3$ is first drawn in the usual manner, together with the rays 1 to 4, inclusive, from any pole O, Fig. 27. Then, by starting at the known

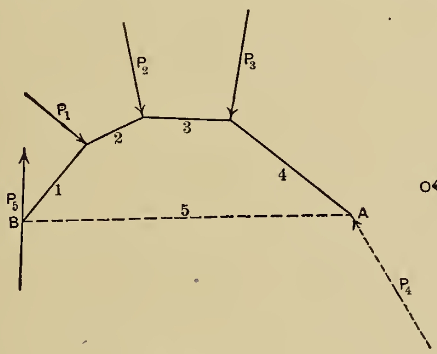


FIG. 26.

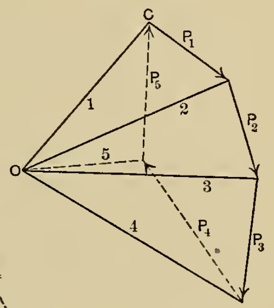


FIG. 27.

point of application A, the sides or bars of the funicular polygon or frame can be drawn parallel to the corresponding rays, so as to fix the point B on the line of action of P_5. The last side 5 of the frame (Fig. 26) will thus be completely determined; and if the ray 5 (Fig. 27) then be drawn parallel to it until it intersects the line drawn through C parallel to the line of action of P_5, the latter force and P_4 will at once become completely known.

This case is one of common occurrence, the forces P_4 and P_5 being in general the reactions or supporting forces

of the structure at the abutments or piers. It is obvious, however, that any other two forces besides the reactions or extreme forces might have been taken as the unknown elements.

If the forces are all parallel, the force polygon $P_1 \ldots P_5$ becomes a straight line, as already explained in Fig. 23, but the process of solution and the various steps in the construction remain unchanged in every particular.

Art. 12.—All Forces Given Except Three, with the Lines of Action of those Three Known.

Let the six forces $P_1 \ldots P_6$ in Fig. 28 represent any system of forces whatever, of which all lines of action are known, but the magnitudes and directions of P_2, P_3,

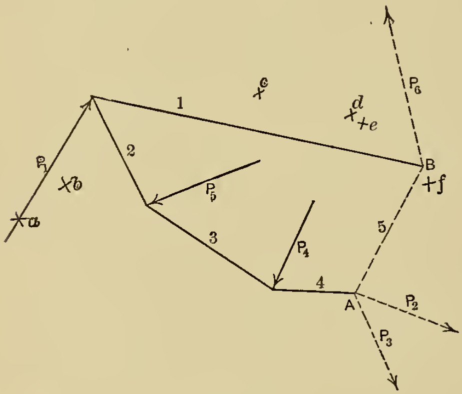


FIG. 28.

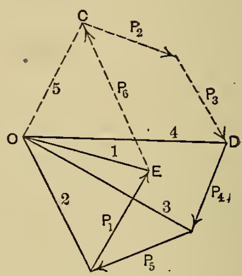


FIG. 29.

and P_6 remain to be determined. The points of application of the forces P_1, P_2, P_3, P_4, P_5, and P_6 are supposed to be a, b, c, d, e, and f respectively, but their lines of action will be prolonged and used for the purposes of this solution. So much of the force polygon as can be constructed from the known forces P_1, P_5, and P_4 is first drawn, and then

the four rays 1, 2, 3, and 4 are drawn from any pole O. The point A (Fig. 28) of intersection of any two lines of action (as those of P_2 and P_3) of the *unknown* forces is next taken as the starting-point of the funicular polygon. The rays drawn to the angles of the partially constructed force polygon will at once enable the sides 4, 3, 2, and 1 of this polygon to be drawn as shown. The points A and B of the remaining side 5 are thus fixed in Fig. 28, and the side itself completed. The ray 5 is then drawn (Fig. 29) until it intersects the line of P_6 drawn from E. The points C and D are thus the starting-points from which P_2 and P_3 are to be drawn until they intersect and so complete the force polygon P_1, P_6, P_2, P_3, P_4, P_5. The latter give the magnitudes and directions of all three of the unknown forces.

By locating an angle of the frame at a point of intersection of the lines of action of two of the unknown forces the sixth side of the frame is reduced to zero and eliminated from consideration, so that the corresponding ray is not needed. Obviously this method cannot be used with parallel forces, for the reason that there will be no finite point of intersection A.

In case the point of intersection A is found to be inaccessible or outside the limits of a convenient diagram, the usual procedure is shown in Fig. 30. CD and AB are the lines of action of P_2 and P_3 respectively, and their intersection is inaccessible. Any point, as E on the adjacent line of action P_4, is taken as the starting-point of the funicular polygon, *the pole O not yet having been determined.* The side 4 of the frame is to be drawn from E to the inaccessible point of intersection. Any points A and C on AB and CD are so taken as to form the triangle ACE with good intersections. From any other point D on CD and at an appropriate distance from C draw DB parallel to CA and then from D and B draw lines parallel to CE

and AE respectively, locating the point F by their intersection. EF produced will then pass through the intersection of AB and CD. EF will then be a portion of the side 4 of the polygonal frame, and the ray 4 must be drawn parallel to it from D in the force polygon of Fig. 29; the pole O may then be located anywhere on that ray. The remaining rays are drawn to the known angles of the force polygon and the construction is made as before. When the point B, Fig. 28, is reached, the remaining side 5 of the frame is to be drawn to the inaccessible point of

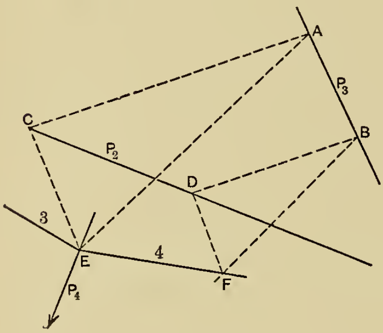


FIG. 30.

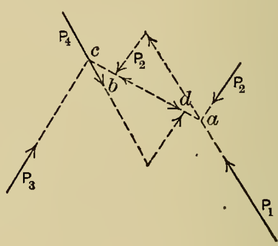


FIG. 31.

intersection of P_2 and P_3 by the process given in connection with Fig. 30.

If there are but four non-parallel forces in the system,* a frequent procedure is that shown in Fig. 31. P_2 is the only completely known force, the lines of action, only, of the others being given. The four forces may be divided into pairs, as P_1 and P_2, and P_3 and P_4, the resultant of each of which pairs must, for equilibrium, be equal and opposite to the resultant of the other. Hence construct

* All systems of the kind treated in this article may be reduced to four non-parallel forces; for all the known forces may be replaced by their resultant, and there are then in addition to this resultant only the three unknown forces. It may be necessary, however, to draw a funicular polygon to determine the point of application of this resultant.

the force triangle P_1, P_2, and the resultant ab. As P_2 is completely given, this can be done and both P_1 and the resultant ab will be determined. Then lay off $cd=ab$ from c and construct the force triangle with P_3 and P_4, which will completely determine those forces and make known all elements of the problem.

The operations of this and the preceding articles are designed to illustrate the general character of the procedures required for the solution of problems by the aid of the force and funicular polygons; they include some of the more important practical cases, although others may arise which are to be treated in the same general manner.

Art. 13.—Stresses in Roof-trusses.

Preliminary.

Roof-trusses are designed to carry, first, their own dead weight; second, the weight of the roof covering on the trusses; third, a snow load; fourth, a wind load acting in a horizontal direction, first from the right, and then from the left; and, fifth, a ceiling or other suspended weights, such as cranes, trackways, shafting, etc.

The weight of the roof-truss itself varies, naturally, with the span length and with the distance between trusses.

Various formulas have been proposed for determining the weights of roof-trusses, and among them is the following, deduced by Milo S. Ketchum, Assoc. M. Am. Soc. C. E., from his experience, for spans up to 150 feet:

$$W=\frac{P}{45}\left(1+\frac{L}{5\sqrt{A}}\right),$$

where W = the weight of truss per square foot of horizontal projection; P=the capacity of the truss in pounds per square foot of horizontal projection; L = the span of

the truss in feet; and A = the distance between trusses in feet. In general, however, for purposes of determining the dead-load stresses, it will be sufficiently accurate to use the following approximate table:

For a Span Length in Feet of	Weight of Truss in Pounds per Square Foot of Ground Surface Covered.
20	2
40	3
60	4
80	5
100	6

Great accuracy cannot be expected from the use of this table; but since, in any event, the greatest stresses are caused by the weight of truss covering, by wind and by snow loads, a small error in assuming the dead weight of a roof-truss is not appreciable. If the error be found appreciable, however, after tentative design, the proper corrections must be made.

Trusses having spans up to 50 or 60 feet usually have one end supported on planed plates; trusses of greater span usually have one end on rollers or on rockers. These precautions are necessary on account of temperature changes. In the case of the roller or rocker end it is usual to assume the reaction at that point perpendicular to the plane on which the rollers move, although on account of friction this condition may not be rigorously exact.

The pitch of the roof is usually given as a ratio of the centre height divided by the span length; it may vary between the limits of 1/2 and 1/5; its more usual value is 1/4.

Snow Load.

The snow load carried by a structure depends not only on the latitude of its location, but also on the pitch of the

roof. It is ordinarily specified as a load in pounds per square foot of horizontal projection. In the vicinity of New York a value of 20 pounds per square foot for flat roofs is usually taken, and this is decreased for roofs with greater pitch. If the pitch is 60° or more, no snow loads need be taken, although a minimum weight of 10 pounds per square foot, due to sleet, is sometimes specified. The highest value for flat roofs in cold climates is 30 pounds per square foot, while for southern latitudes the snow load disappears.

Wind Load.

In treating horizontal wind loads on roof-trusses, the component normal to the slope of the roof is usually taken, the component parallel to the slope of the roof being neglected. The intensity of horizontal wind pressure on a vertical surface is generally specified at 30 pounds per square foot.*

The normal pressure on a roof due to horizontal wind force is not usually found by its simple resolution into two rectangular components, but is determined by means of an empiric formula based upon experimental work. Two such formulæ are in common use, one by Hutton, given in eq. (1), and the other by Duchemin, given in eq. (2).

$$P_n = P \sin \alpha^{1.842 \cos \alpha - 1}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P_n = P \frac{2 \sin \alpha}{1 + \sin^2 \alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where P_n represents the normal component, P the horizontal force, and α the angle between the roof surface and a horizontal plane.

---

* For a more detailed statement concerning wind pressures, see Supplementary Report by Capt. W. H. Bixby in the Report of the Board of Engineer Officers, as to Maximum Span Practicable for Suspension Bridges, Washington, 1894.

The following table furnishes the values of the ratios of P_n to P for the various slopes of roof indicated:

VALUES OF $\frac{P_n}{P}$.

Slope of Roof in Degrees.	Duchemin.	Hutton.
0	0.00	0.00
10	0.34	0.24
20	0.61	0.46
30	0.80	0.66
40	0.91	0.84
50	0.97	0.95
60	0.99	1.00

Roof Covering.

The roof covering rests upon longitudinal purlins carried by the truss, usually at its panel points. If the purlins are not placed at the apices or panel points, bending is caused in the chord members. Such stress conditions should ordinarily be avoided for simplicity.

The amount of load carried to any panel point is immediately determined by noting the area of surface and the load per square foot carried by each purlin. This area, or its horizontal and vertical projection determines also the snow load and the wind load carried at any panel point.

The weight of roof covering varies with the different materials of which it is composed, but it may be closely estimated in advance.

The weights of roof coverings may be approximately assumed as follows:

Iron Sheets.—The weight of iron or steel sheeting depends on the gauge thickness and whether the sheets are flat or corrugated. The exact weight may always be found in manufacturers' handbooks; it varies from 1 to 3 pounds per square foot for the thicknesses ordinarily employed.

Felt, Pitch, and Slag or Gravel Roofing.—This combination of materials may weigh from 8 to 10 pounds per square foot, depending generally on the number of thicknesses of felt.

Slate.—Slate usually weighs from 7 to 9 pounds per square foot of roof.

Tile.—Terra-cotta tile 1 inch thick weighs about 6 pounds per square foot.

Tin.—Tin, without sheathing, weighs from 1 to $1\frac{1}{2}$ pounds per square foot.

None of these weights includes weights of purlins or sheathing.

Wooden Coverings.—The weight of wooden roof coverings may be estimated by assuming the weight of wood at 4 pounds per foot B. M.

The exact weight of roof covering, including sheathing, purlins, bracing, gutters, ventilators, etc., must be calculated for each individual problem. In a similar way, all suspended weights must be determined before the stresses can be computed.

The total loads carried by any truss having then been estimated, the determination of the stresses in the members of the structure is the next procedure. This determination may be made by combining the various classes of loads and proceeding with resultants or by treating each class separately and subsequently combining the stresses so found for each member. The method of procedure will be obvious in each case.

Art. 14.—Stresses in a Roof-truss, Both Ends Fastened.

A common form of roof-truss is illustrated in Fig. 32, in which the span is 40 feet, the rise of the peak 10 feet, and the distance between trusses 12 feet. The roof covering for this truss is estimated to weigh 10 pounds

per square foot, and since the total exposed surface between two neighboring trusses is 45 feet long by 12 feet wide, the total roof covering weighs $45 \times 12 \times 10 = 5400$ pounds. The dead weight of the truss itself has been estimated at 3 pounds per square foot of horizontal projection. The area of horizontal projection is $40 \times 12 = 480$ square feet;

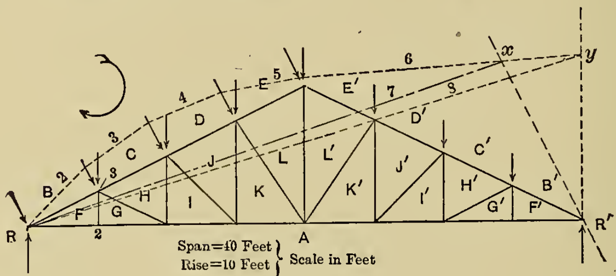


FIG. 32.

the dead weight of one truss is, therefore, 1440 pounds, and the total dead weight $5400 + 1440 = 6840$ pounds. This may be supposed equally divided among the upper panel points, the end points carrying but half a panel load. Each panel load is, therefore, 855 pounds. It is obvious that the end panel loads, if vertical, need not be considered as a factor in causing stresses in the structure, since these are carried directly by the abutments. Vertical end panel loads will, therefore, not be considered in the cases to be discussed.

The dead-load stress diagram is shown in Fig. 33; its construction is as follows:

Since the load is symmetrical, each reaction is equal to half the total load on a truss. The line *ab* is therefore drawn vertically upward and equal by scale to the left-hand reaction. The stresses in the two members meeting at the left-hand abutment may at once be found, as there are only two unknown quantities. The lines *bf* and *fa* are, therefore, drawn parallel to the corresponding lines in Fig. 32 through the points *b* and *a* to their intersection at *f*, thus determining the stresses *bf* and *fa*. The circular

arrow in the upper part of Fig. 32 indicates the direction in which the forces are read about any panel point, and

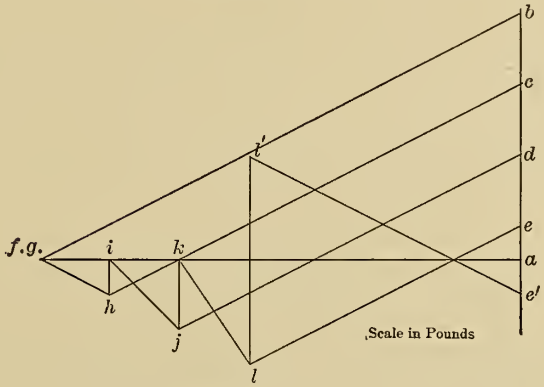


FIG. 33.

this order must be carefully observed. By transferring the direction of *bf* from Fig. 33 to Fig. 32, it will be seen that the stress acts toward the panel point, indicating compression. Similarly, *fa* acts away from the panel point, indicating tension. Hereafter the explanation determining the sign of the stress in any member need not be given.

Panel point 2 is next to be considered. With the distribution of load assumed there will be no stress *fg*, and *fa* must evidently be equal to *ga*.

Panel point 3 is next in order. The lines *ch* and *hg* are drawn parallel to their respective lines of action in Fig. 32. The remaining construction is precisely similar in character and needs no further explanation for the half of truss under consideration.

Fig. 33 represents the stresses for the left half of the truss only; since the structure is symmetrical the members in the right half carry identically the same stresses as the corresponding left-hand members. The values of the stresses as scaled from Fig. 33 are given in Table I. The

positive sign indicates tensile stress, and the negative sign compression.

A separate diagram for snow loads over the entire roof need not be drawn for roof-trusses of this character, since it is at once evident that the snow-load stresses may be found as a simple ratio of the dead-load stresses. If the snow load be taken at 20 pounds per square foot of horizontal projection, the ratio of the snow-load stresses to those due to the dead load will be

$$\frac{20(40\times 12)}{6840}=1.4.$$

The values of the stresses obtained from this ratio are given in Table I.

The intensity of the wind load, which acts on one side of the roof only, has been taken at 30 pounds per square foot of vertical surface. By means of Hutton's formula, the normal component for this truss will be $30\times .59=17$ pounds per square foot, since the angle of inclination of the roof surface is $26° 30'$. The distance between upper panel points is 5.63 feet. Each panel load will therefore be $5.63\times 12\times 17=1150$ pounds, acting normally to the roof surface. The wind panel loads at the peak and at the abutment will, of course, be only half as great, or 575 pounds.

The wind stresses in the truss will be found on the assumption that the reactions at the two ends of the truss are parallel to each other and to the direction of the wind loads. In order to determine the amounts of these reactions, the funicular polygon must be employed. The loads are first laid down to scale, as shown in Fig. 34. Any pole, as O, is chosen and the rays 1, 2 . . . 6 drawn and transferred to Fig. 32, the polygon being started at the left-hand reaction point with ray 2. Since this polygon

must close for equilibrium, the missing ray 7 lying between the lines of action of the reactions may at once be drawn. If this ray be transferred to Fig. 34, it will determine at once the point a and give the reactions $r'a$ and ar. The stresses in the structure for the wind load may then be

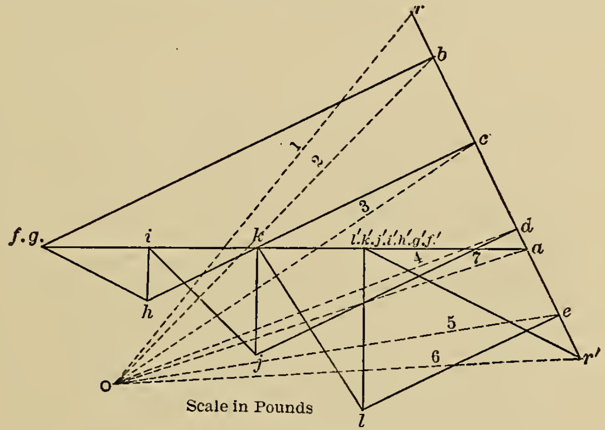


FIG. 34.

found exactly as in the case of the dead load, the left-hand reaction point being again the first point treated. The force polygon, Fig. 34, thus being constructed precisely as was Fig. 33, requires no further explanation.

Attention is called to the fact that the stresses in all the web members of the right half of the truss are zero for the direction of the wind shown; but since the wind may be taken to blow in either direction, the stresses in the right-hand members will be found to be the same as in the left-hand members, if the direction of the wind be reversed. The values of the wind stresses are given in Table I. The final stresses for which the members of this truss must be designed are those found by combining the results due to dead, snow, and wind loads; they are shown in the last column of the table. Since no member suffers

reversal of stress, the sum of all the stresses, viz., for dead, for snow, and for wind loads, determines the maximum stress existing in each member.

If, however, in the judgment of the designer, the wind stresses and the snow stresses might never occur at the same time, only the sum of those stresses which might act simultaneously should be taken.

TABLE I.

	Member.	Dead-load Stress.	Snow-load Stress.	Wind-load Stress.	Maximum Stress.
Upper chord	*BF*	−10,500	−14,700	−5,200	−30,400
	CH	− 5,460	− 7,650	−4,350	−17,460
	DJ	− 4,600	− 6,450	−3,450	−14,500
	EL	− 3,650	− 5,100	−2,600	−11,350
Lower chord	*AF*	+ 5,750	+ 8,050	+5,800	+19,600
	AG	+ 5,750	+ 8,050	+5,800	+19,600
	AI	+ 4,950	+ 6,930	+4,500	+16,380
	AK	+ 4,100	+ 5,750	+3,200	+13,050
Web members	*FG*	0	0	0	0
	GH	− 900	− 1,260	−1,400	− 3,560
	HI	+ 420	+ 590	+ 650	+ 1,660
	IJ	− 1,200	− 1,700	−1,850	− 4,750
	JK	+ 850	+ 1,200	+1,300	+ 3,350
	KL	− 1,500	− 2,100	−2,300	− 5,900
	LL'	− 2,480	− 3,480	−1,950	− 7,910

Art. 15.—Stresses in a Roof-truss, One End on Rollers.

Let the truss of the previous article have one end on rollers and let the wind stresses be determined.

It will be unnecessary to repeat the determination of the stresses caused by vertical loads, since they are in no way affected by the roller end. The values shown in Table II for the dead and snow loads are the same as those in Table I.

The lines of action and the amounts of the reactions for the wind loads must first be found. In determining the reactions, the problem is simply the general case of a

series of forces in equilibrium, all known except two, of which the point of application of the one and the line of action and direction of the other are known. Assuming the direction of the wind from left to right and the roller end at R', the funicular polygon (Fig. 32) 2, 3 . . . 6 is drawn as shown, the only precaution being that the polygon must be started at the point of application of the unknown force R. The pole O chosen, and the rays 2, 3 . . . 6 used, are shown in Fig. 35. The ray 8 is found (Fig. 32) by drawing it from y so as to close the figure. The direction of ray 8

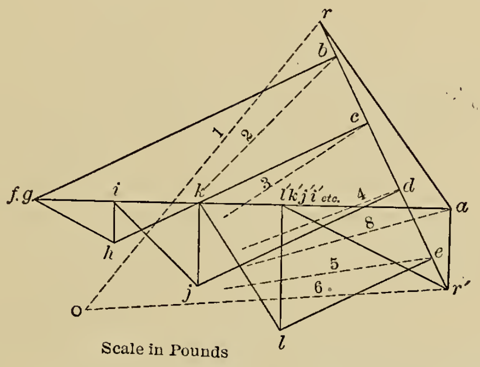


FIG. 35.—Stress Diagram; Right End on Rollers.

is then transferred to Fig. 35, where it defines the point a, which determines at once the right-hand reaction $r'a$ and the left-hand reaction ar. The further construction of the stress diagram is precisely the same as was followed for Fig. 34.

There now remains the construction of a diagram for the wind blowing from left to right, and with the same end on rollers. But this problem may be treated more advantageously by retaining, as in Fig. 35, the direction of the wind from left to right, and transposing the roller end from the right end to the left end. The stresses found from such a diagram (Fig. 36) must, however, apply to the members placed in a symmetrical position about the centre

line of the truss; that is, the letters designating the stresses in Fig. 36 are all primed letters. This method of treatment evidently does not apply to unsymmetrical trusses.

The reactions for this case may be found most simply in the following manner: It is evident that the vertical components of the two reactions are the same, no matter which end of the truss is on rollers; therefore, in Fig. 36, *r′a*

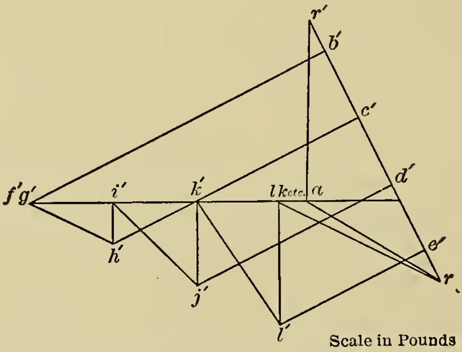


FIG. 36.—Stress Diagram; Left End on Rollers.

may at once be drawn from Fig. 35, since it is merely the vertical component of *ra* there determined. It is only necessary then to connect the points *r* and *a* (Fig. 36) to obtain the reaction *ra*. After these reactions are determined the stress diagram is to be constructed precisely as before.

The final stresses in the structure for all possible loads are shown in Table II. The final column shows the range of stress to which each member may be subjected and for which it must be designed; it is to be observed that the dead-load stress must always appear in that range, the other stresses being included only as the judgment of the designer may indicate. It is evident also that in the range of stress for any member there must appear but one value of the wind stress, for the wind cannot act simultaneously in both directions.

TABLE II.

	Member.	Dead-load Stress.	Snow-load Stress.	Wind-load Stress; Wind Left to Right.	Wind-load Stress; Wind Right to Left.	Range of Stress.
Upper chord	$BF=B'F'$	−10,500	−14,700	−5,200	−5,200	−10,500 / −30,400
	$CH=C'H'$	− 5,460	− 7,650	−4,350	−4,350	− 5,460 / −17,460
	$DJ=D'J'$	− 4,600	− 6,450	−3,450	−3,450	− 4,600 / −14,500
	$EL=E'L'$	− 3,650	− 5,100	−2,600	−2,600	− 3,650 / −11,350
Lower chord	AF	+ 5,750	+ 8,050	+6,500	+400	+ 5,750 / +20,300
	AG	+ 5,750	+ 8,050	+6,500	+ 400	+ 5,750 / +20,300
	AI	+ 4,950	+ 6,930	+5,200	+ 400	+ 4,950 / +17,080
	AK	+ 4,100	+ 5,750	+3,900	+ 400	+ 4,100 / +13,750
	AK'	+ 4,100	+ 5,750	+2,600	+1,700	+ 4,100 / +12,450
	AI'	+ 4,950	+ 6,930	+2,600	+3,000	+ 4,950 / +14,880
	AG'	+ 5,750	+ 8,050	+2,600	+4,300	+ 5,750 / +18,100
	AF'	+ 5,750	+ 8,050	+2,600	+4,300	+ 5,750 / +18,100
Web members	FG	0	0	0	0	0
	GH	− 900	− 1,260	−1,400	0	− 900 / − 3,560
	HI	+ 420	+ 590	+ 650	0	+ 420 / + 1,660
	IJ	− 1,200	− 1,700	−1,850	0	− 1,200 / − 4,750
	JK	+ 850	+ 1,200	+1,300	0	+ 850 / + 3,350
	KL	− 1,500	− 2,100	−2,300	0	− 1,500 / − 5,900
	LL'	− 2,480	− 3,480	−1,950	−1,950	− 2,480 / − 7,910
	$K'L'$	− 1,500	− 2,100	0	−2,300	Same as KL
	$J'K'$	+ 850	+ 1,200	0	+1,300	" " JK
	$I'J'$	− 1,200	− 1,700	0	−1,850	" " IJ
	$H'I'$	+ 420	+ 590	0	+ 650	" " HI
	$G'H'$	− 900	− 1,260	0	−1,400	" " GH
	$F'G'$	0	0	0	0	" " FG

Comparison of Tables I and II shows that for one end on rollers no stresses differ from those found with both

ends fastened, except in the lower chord. Table II furnishes lower values for the lower chord members near the fixed point of support, and higher values at the roller end. This statement as to variations in stress for the two methods of end support is not a general one, although it applies in this case, where the entire lower chord is in one straight line.

Counterbraces.

If the range of stress to which a web member is subjected includes values of both tension and compression, that member must be designed to resist both; it is then said to be counterbraced. If such a member is constructed that it can resist tension only, provision for the compressive stress may be made by inserting in the same panel, and crossing the original member, an additional member known as a counterbrace. It will be found that this member will be stressed in tension when the compressive stress in the original member causes the latter to be useless; the stresses in the members of the structure affected by this new condition may then be found by assuming that the counterbrace is the main member, and that the original member does not exist,

Art. 16.—Fink Roof-truss.

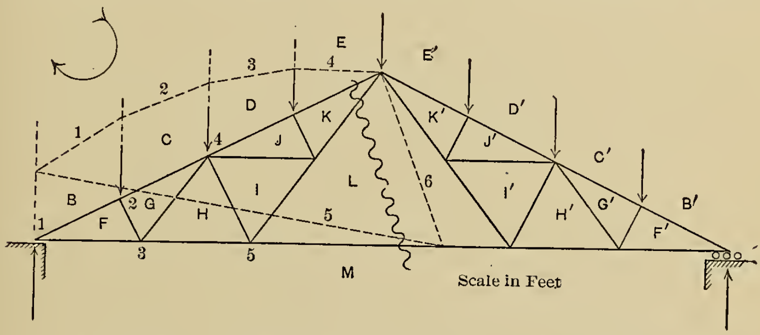
Fig. 37 represents an application of the Fink truss to roof construction. It presents a slightly more complex problem than the truss illustrated in the previous articles.

The following data are assumed:

Span = 60 feet;
Rise of peak = 15 feet;
Distance between trusses = 14 feet.

The roof covering, including purlins and wind bracings, is taken at 12 pounds per square foot of inclined surface, and

the dead weight of one truss at 3.8 pounds per square foot of *horizontal* projection. A panel load will then be found to be 1175 + 400 = 1575 pounds. The stress diagram (Fig. 38) is again begun by drawing *mb* vertically upward as the left-hand reaction and determining the stresses in *bf* and *fm*; the direction of rotation about any point being indicated by the circular arrow (Fig. 37.) Panel points 2 and 3



Span = 60 feet. Rise = 15 feet. Distance between trusses = 14 feet.

Dead Weights Assumed.

Roof covering including purlins and wind-bracing = 12 pounds per sq. ft. surface.
Truss = 3.8 pounds per sq. ft. horizontal projection.
Snow load = 30 lbs. per sq. ft. horizontal projection.
Wind load = 40 lbs. per sq. ft. vertical projection.

FIG. 37.

may then be treated in precisely the way already explained, but it will be found that at both panel points 4 and 5 there will be three unknown quantities instead of two, and that recourse must be had to some other method than the use of the simple force diagram. A section is, therefore, passed through the truss, cutting, as shown, the members *EK*, *KL*, and *LM*. The stresses in the members cut hold in equilibrium all of the external forces on one or the other side of the section. In this case the left-hand forces will be considered.

The stresses in these three members may be determined by means of the general principles already explained, for

the problem is simply that of a set of forces in equilibrium, all known except three, the lines of action of the latter being given. The funicular polygon is, therefore, drawn, the only precaution being that the polygon Fig. 37 must start at the intersection of two of the unknown forces. In this case the peak was chosen as the starting point, being the intersection of the members *EK* and *KL*. The rays 4, 3, 2, 1, and 5 were then drawn. As the forces are in equilibrium the polygon must close, and the closing ray 6 may then be drawn and transferred from Fig. 37 to Fig. 38, where it immediately defines the point *m* and determines

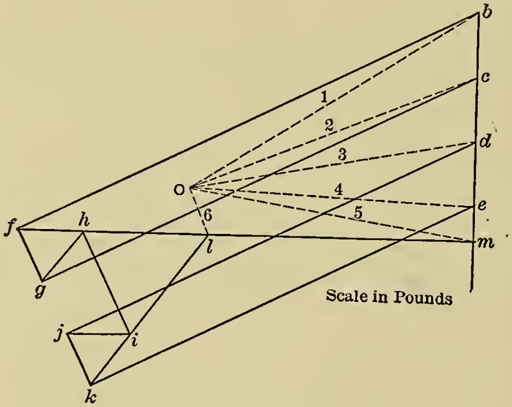


FIG. 38.

the stress in the member *ML*. Knowing then the stress in *LM*, panel point 5 may be treated in the usual manner, as the stresses in the members *HI* and *IL* are the only unknown forces. Panel point 4 may next be treated in the usual manner and all the stresses be determined without further explanation.

A more simple solution of this problem is possible by determining the stress in the member *LM* by taking moments about the peak and then inserting its value in the force

diagram Fig. 38. The graphic solution for the remaining stresses can then proceed in the usual way. The moment equation for the peak as a centre would then be as follows:

$$MB \times 30 - BC \times 22\tfrac{1}{2} - CD \times 15 - DE \times 7\tfrac{1}{2} - LM \times 15 = 0,$$

or

$$5510 \times 30 - 1575(22\tfrac{1}{2} + 15 + 7\tfrac{1}{2}) - LM \times 15 = 0,$$

and the stress in the member LM would be

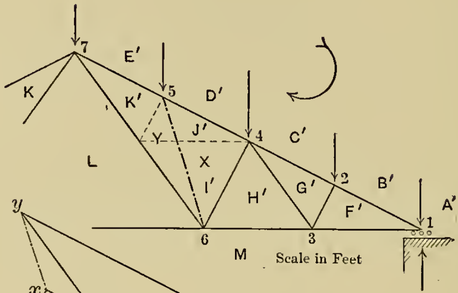
$$LM = +6300 \text{ pounds.}$$

This, of course, should check the value of the stress found graphically.

It is proper to explain also one other method of solution of the Fink truss, as illustrated in Figs. 39 and 40. It is the method of substitution of diagonals, and is shown applied to the right half of the truss. If the members $K'J'$ and $J'I'$ be removed from the structure and be replaced temporarily by the member YX, the stress in the member LM may be found by the methods of the ordinary force polygon, the panel points 1, 2, 3, 4, 5, and 6 being taken in the order named, since at no panel point will there be more than two unknown forces. This substitution of the diagonal YX does not affect in any way the stress in the member LM, for the stress in that member cannot be affected by changes in any other panel of the character indicated. In fact, the portion of the truss defined by panel points 7, 6, and 1 might be replaced by any rigid mass. Having, therefore, found the stress in the member LM by this method of substitution, the original web members $K'J'$ and $J'I'$ may again be replaced and their stresses found in the usual manner, without further diffi-

culty. No further use is made of the dotted line xy in Fig. 40.

FIG. 39.



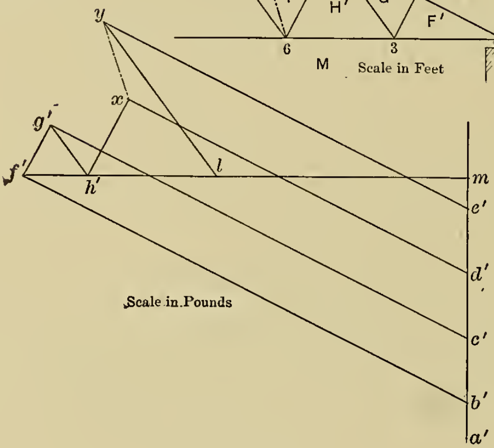


FIG. 40.

The treatment of the wind stresses in a Fink truss need not be considered here in detail, as the methods to be used are the same as those employed in the preceding articles.

Art. 17.—Unsymmetrical Trusses.

The treatment of unsymmetrical trusses is precisely the same as for those that are symmetrical. Proper consideration should, however, be accorded to the determination of the panel loads, since these are in general no longer equal, and it will usually be convenient to determine the

reactions of the truss by means of the funicular polygon. In Fig. 41, which represents an unsymmetrical truss carrying at the upper panel points the weights shown, the reactions *es* and *sa* were determined by means of the lower funicular polygon 1, 2, 3, 4, 5, 6, the directions of these

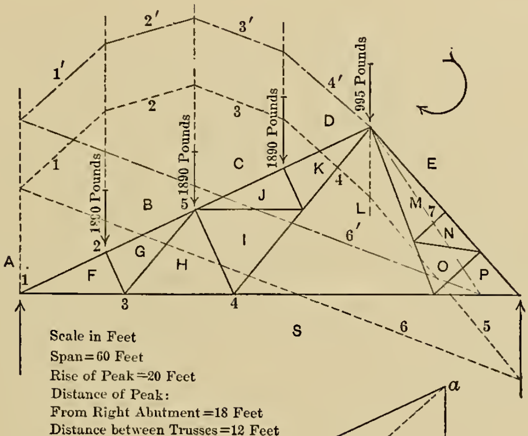


FIG. 41.

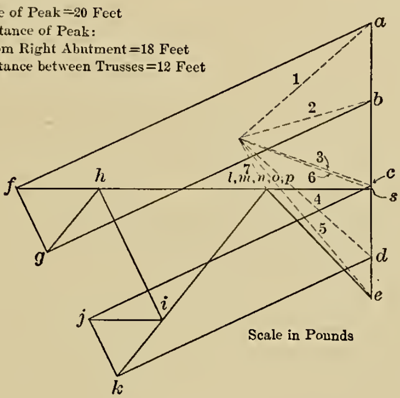


FIG. 42.

rays being obtained from Fig. 42. The transference of ray 6 from Fig. 41 to Fig. 42 then fixes the point *s* which determines the amounts of the two reactions. The finding of the stresses presents no difficulties until panel point 4 or 5 is reached, when one of the constructions of the previous

article must be employed. In this case the funicular polygon becomes 4′, 3′, 2′, 1′, 6′, the point of beginning being chosen at the peak. The ray 7 becomes the closing line, and being transferred to Fig. 42 determines the position of the point *l*. The remaining stresses may then be found in the usual manner.

The preceding constructions are all that are necessary to determine the fixed or dead-load stresses in any simply supported structure, and they are immediately available for finding the dead-load stresses in railroad or highway bridge-trusses. They are also applicable for finding the dead-load stresses in the open arms of swing bridges. These constructions are equally applicable to the dead-load stresses in both cantilever bridges and three-hinged arches, but the treatment of the fixed-load stresses in such structures will be considered at the same time with the moving loads.

Art. 18.—Bending of Supporting Columns of Roofs.

The roof-trusses of the preceding articles have been treated as resting on supports or walls capable of resisting the horizontal forces acting against them, but the stresses in which do not affect the stresses in the truss members. If, however, the roof-truss be a part of the transverse bent in the building (Fig. 43) connected to the vertical columns by knee-brackets, then the bending stresses produced in the columns cause additional stresses in the members of the truss.

The dead loads and snow loads resting on the roof are vertical forces that cause no bending stresses in the columns and no stresses in the knee-brackets, provided the deformation of the truss itself be neglected. If the deflections of the truss are considerable, however, additional stresses will

be produced in the knee-brackets and columns, and also in the members of the truss; generally such deflections are not considered. It is evident, then, that the methods of the previous articles when treating vertical loads apply also to trusses mounted on columns. Non-vertical loads

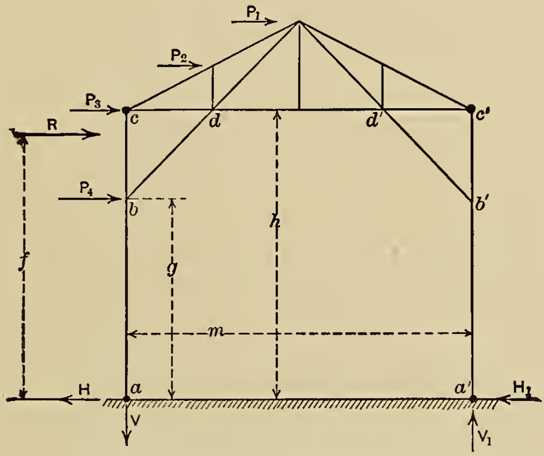


FIG. 43.—Posts Hinged at Top and Base.

require further consideration. Such loads may include not only wind loads, but loads caused by shafting, hoists, cranes, etc. In this article horizontal wind loads only will be considered, but the treatment will be general, so that it may be applied to loads of any character which may be resolved into vertical and horizontal components.

The wind forces acting against the vertical side of the building and causing flexure in the posts must be included in the loading. Since that surface is usually much greater than that exposed by the roof-truss itself, it is usual in the consideration of this type of roof-truss to treat the wind load on the truss as horizontal, instead of normal to the surface, as previously, but the method is equally applicable to normal loads.

Fig. 43 illustrates a roof-truss mounted on the posts *ac* and *a'c'* and connected to them by the knee-brackets *bd* and *b'd'*. It is generally assumed that the upper ends *c* and *c'* of the posts or columns are hinged to the truss so that the latter is free to turn about those points; this involves the condition that there is no bending moment in the posts at those points. Such a condition may never be realized in practice, but it is on the side of safety. The fastening of the bases *a* and *a'* of the columns admits, however, of two distinct methods of treatment:

Case I. The columns may be considered hinged at *a* and *a'* and free to turn about those points.

Case II. The columns may be considered firmly fastened at *a* and *a'*. In that case the anchorage must provide for the bending moments existing at those points.

In practice it is doubtful if the conditions of either Case I or Case II are ever exactly fulfilled. The true condition lies probably between the limits indicated, but it will always be found on the side of safety, as far as the truss or post stresses are concerned, to treat all problems by the methods of Case I. It is to be observed that Case I requires no anchorage in the base to resist bending, while Case II requires an anchorage.

Case I.

The problem is most easily solved by three distinct operations: (1) Determining the forces causing bending stresses in the columns; (2) transferring the loads created by these bending stresses to the panel points of the framed structure, and (3) finding the stresses in the members of the framed structure for these loads and all other external loads acting upon it.

The loads to be treated are shown in Fig. 43 as P_1, P_2, P_3, and P_4; under certain conditions it will prove convenient

to employ their resultant R acting at a distance f above the surface. It will first be necessary to find the rectangular components H, V, H_1, and V_1 of the reactions acting at a and a', the bases of the posts. In the following treatment H will always be taken equal to H_1; it is perhaps possible, by an analysis based on the deflections of the structure, to obtain more accurate values, but such refinement is unnecessary. In the present case, therefore,

$$H = H_1 = \frac{R}{2}. \qquad \ldots \ldots \ldots \quad (1)$$

Taking moments of all the external forces about a',

$$R \cdot f - V \cdot m = 0 \quad \text{or} \quad V = \frac{R \cdot f}{m} = -V_1. \quad \ldots \quad (2)$$

The structure may now be divided into the three parts shown in Figs. 44, 45, and 46. Figs. 44 and 46 represent the forces acting on the posts ac and $a'c'$ respectively, while Fig. 45 shows all the panel loads acting on the statically determinate framework there shown. The forces causing flexure in the posts are also loads acting on the truss, but they are exactly reversed in direction. This is evident, for the forces Q_1, Q_2, Q_3, and Q_4 are simply used as temporary devices to indicate forces acting on the posts and must be eliminated from the problem by considering them as loads causing stresses in the triangular framework. Fig. 45 therefore shows, as loads at the panel points, the forces P_1, P_2, P_3, and P_4 and also the forces Q_1, Q_2, Q_3, and Q_4, every one of the latter forces being exactly equal but opposite in direction to the similarly lettered forces of Figs. 44 and 46.

Since the conditions of the problem state that the

posts are hinged at the bases, each may be treated as a simply supported beam carrying a concentrated load Q_2 or

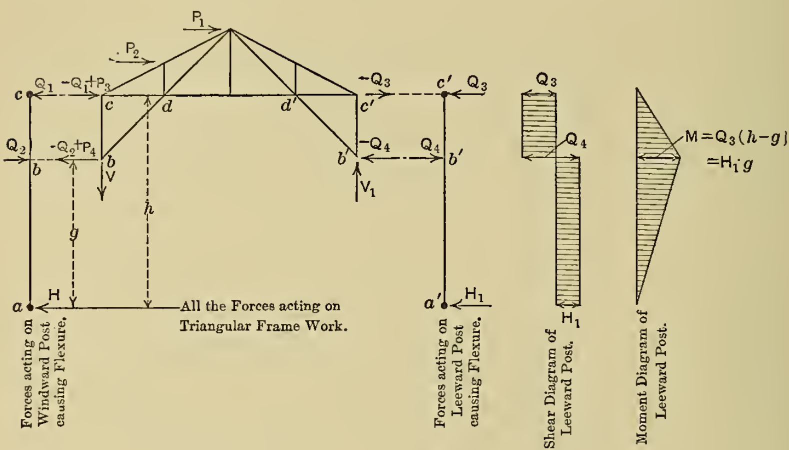


FIG. 44. FIG. 45. FIG. 46. FIG. 47. FIG. 48.

Q_4 at b or b', the points of attachment of the knee-brackets. Treating post ac and taking moments about b,

$$H \cdot g = Q_1(h-g).$$

$$Q_1 = \frac{H \cdot g}{h-g}, \quad \ldots \ldots \ldots \quad (3)$$

and from the relation that $\Sigma H = 0$,

$$Q_2 = H + Q_1. \quad \ldots \ldots \ldots \quad (4)$$

Eqs. (3) and (4) furnish the values of the forces Q_1 and Q_2, and in precisely the same way there may be found the values of Q_3 and Q_4. The shear and bending moment existing at every point in the post $a'c'$ are represented graphically in Figs. 47 and 48. The column section must

be capable of carrying these shearing and bending stresses in addition to its direct stress of tension or compression.*

The second step in the analysis is to transfer in the proper manner to Fig. 45 the forces Q_1, Q_2, Q_3, and Q_4 and to unite them with the loads P whenever they act at the same panel points. For instance, at panel point c the load is $-Q_1+P_3$, whereas at c' it is only $-Q_3$; similarly at b the load is $-Q_2+P_4$, whereas at b' it is only $-Q_4$. It is also necessary to insert as loads on the structure the vertical forces V and $-V_1$ at the points b and b', which are also the values of the direct stresses in the posts ab and $a'b'$, for they are the vertical components of the reactions transferred directly from the bases a and a'.

The stresses in the members of the framework of Fig. 45 may then be found at once by a single stress diagram which it is not necessary to reproduce. It will be found that the portions bc and $b'c'$ of the posts sustain different stresses from the lower portions ab and $a'b'$ of the same members.

After having obtained the direct stresses in the members bc and $b'c'$, these must be united in the proper manner with the bending stresses previously found. It is seen that the preceding treatment is perfectly general and may be applied to other than horizontal forces.

CASE II.

Fig. 49 illustrates the second method of treatment in which the posts are assumed to be rigidly connected at the bases a and a'. In consequence of this rigid connection each post acts similarly to a beam (Fig. 50) which

---

* For the design of members subjected to combined stresses, such as flexure and tension or flexure and compression, see Burr's "Resistance of Materials," p. 164.

is rigidly fastened at one end a, simply supported at the other end c, and which carries a weight at b acting at right angles to the axis of the beam. The post is represented

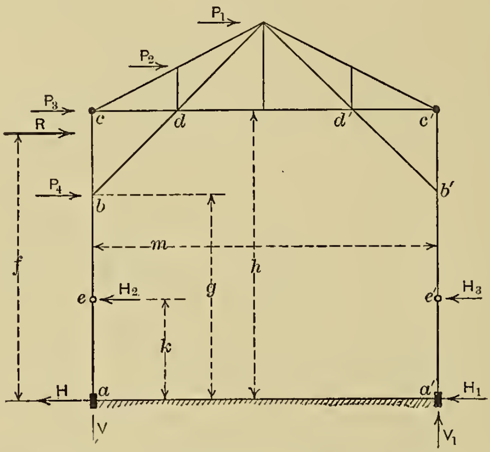


FIG. 49.—Posts Hinged at Top and Fixed at Base. Points of Contraflexure at e and e'.

in Fig. 50 as a horizontal beam, and it contains at some point in its length a point of contraflexure, or point of no bending, as at e. If it be assumed that the points b and c deflect equal amounts bb'' and cc'', the following analysis will locate the point of contraflexure.

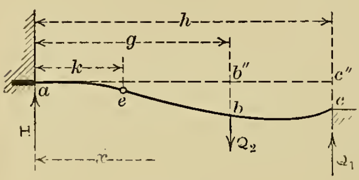


FIG. 50.

The bending moment to which the portion ab of the beam is subjected at any point distant x from a may be represented as follows:

$$M_{ab} = Q_1(h-x) - Q_2(g-x) = EI\frac{d^2y}{dx^2}. \quad . \quad . \quad (5)$$

The last member in eq. (5) is derived from the theory of flexure, in which the external bending moment at any section is equal to $EI\frac{d^2y}{dx^2}$, where E is the coefficient of elasticity, I is the moment of inertia of the cross-section of the beam, and y and x are the respective coordinates of any point. Integrating eq. (5), the tangent of inclination at any point of the length ab, multiplied by EI, will be

$$EI\frac{dy}{dx}=Q_1\left(hx-\frac{x^2}{2}\right)-Q_2\left(gx-\frac{x^2}{2}\right)+(C=0). \quad . \quad . \quad (6)$$

$$C=0, \text{ since } \frac{dy}{dx}=0 \text{ when } x=0.$$

For the portion bc of the span the moment is

$$M_{bc}=Q_1(h-x)=EI\frac{d^2y}{dx^2}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Integrating as before,

$$EI\frac{dy}{dx}=Q_1\left(hx-\frac{x^2}{2}\right)+C_1. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

C_1 may be evaluated, for $EI\frac{dy}{dx}$ in eq. (8) has the same value as in eq. (6) at the point b where $x=g$.

Therefore $$C_1=-\frac{Q_2g^2}{2}$$

and eq. (8) becomes

$$EI\frac{dy}{dx}=Q_1\left(hx-\frac{x^2}{2}\right)-\frac{Q_2g^2}{2}. \quad . \quad . \quad . \quad . \quad (9)$$

Integrating again,

$$EIy=Q_1\left(\frac{hx^2}{2}-\frac{x^3}{6}\right)-\frac{Q_2g^2x}{2}+C_2. \quad . \quad . \quad . \quad (10)$$

y in eq. (10) represents the deflection of any point of the beam between b and c. When $x=g$ or h, the deflections are equal, according to the premises of the problem; that is,

$$Q_1\left(\frac{hg^2}{2}-\frac{g^3}{6}\right)-\frac{Q_2g^3}{2}+C_2=Q_1\left(\frac{h^3}{2}-\frac{h^3}{6}\right)-\frac{Q_2g^2h}{2}+C_2. \quad (11)$$

Cancelling C_2 from each member, the following relation may be obtained:

$$\frac{Q_1}{Q_2}=\frac{3g^2}{-g^2+2hg+2h^2}. \quad . \quad . \quad . \quad . \quad (12)$$

Since the moment of the external forces is zero about the point of contraflexure,

$$Q_1(h-k)=Q_2(g-k). \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\frac{Q_1}{Q_2}=\frac{g-k}{h-k}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Subtracting eq. (14) from eq. (12), and solving for k, there is obtained

$$k=\frac{g}{2}\left(\frac{2h+g}{h+2g}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Table I furnishes the values of k in terms of h for various values of g in terms of h.

TABLE I.

When g equals	Then k equals
h	$.50g$
$.9h$	$.52g$
$.8h$	$.54g$
$.7h$	$.56g$
$.6h$	$.59g$
$.5h$	$.62g$
$.4h$	$.67g$
$.3h$	$.73g$
$.2h$	$.75g$

Since g is seldom less than $.5h$, and since moreover the effect of the vertical loading on the column has not been included in the treatment, Table I shows that it will prove sufficiently accurate to take the point of contraflexure midway between a and b.

Having determined the position of the points of contraflexure, e and e', in Figs. 51 and 53, the forces causing flexure in the post may at once be found. The windward post only will be treated.

As in Case I, $H=H_1=\frac{R}{2}$. Since the moment about e equals zero,

$$Q_1(h-k)=Q_2(g-k);$$

also, $$Q_1=Q_2-H \text{ from } \Sigma H=0;$$

therefore $$Q_2=\frac{H(h-k)}{h-g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

or precisely the same value as if the post had a length ec and the force H were acting at e, the latter point being considered a hinge support; that is, as far as the bending in ec is concerned, the force H at a might be replaced by

an equal force H_2 acting at e. The moment at the base a is then $H_2 \cdot k = H \cdot k$. In the same way a force $H_3 = H_1$ may be placed at e'.

The vertical components of the reactions may now be found. Taking moments of the external forces about a,

$$R \cdot f - H_2 \cdot k - H_3 \cdot k - V \cdot m = 0. \quad . \quad . \quad . \quad (17)$$

But $H_2 + H_3 = R$; therefore eq. (17) may take the form

$$R(f-k) = V \cdot m = -V_1 m. \quad . \quad . \quad . \quad . \quad (18)$$

The values of V and V_1 are therefore precisely the same as if the posts were hinged at the points e and e', and it is

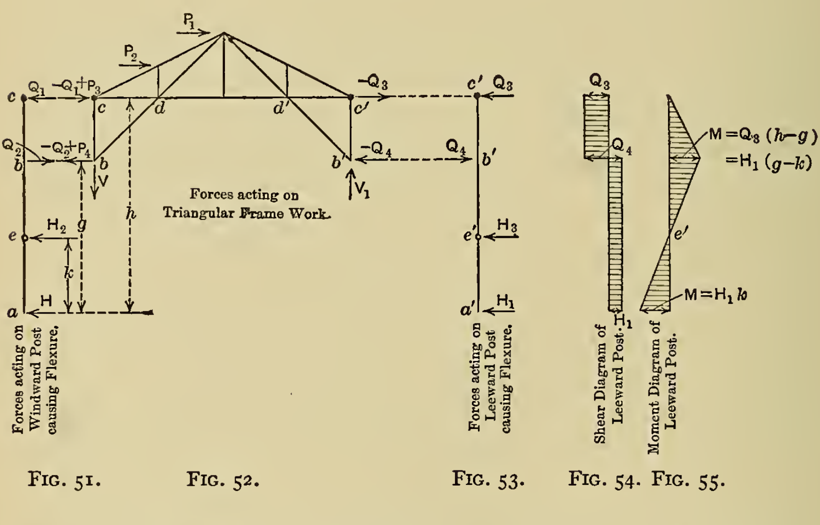


FIG. 51. FIG. 52. FIG. 53. FIG. 54. FIG. 55.

seen that Case II is a repetition of Case I if in the former the posts be shortened by an amount k.

Case II therefore requires no further treatment, since from this point it is the same as Case I; Figs. 51, 52,

and 53 show the panel loads and other forces acting on the members.

It should be noted, however, that the force P_4 in Case II has not the same value as in Case I, for in Case II it is only the wind load acting on the surface between points e and c, whereas it previously was the wind load acting between a and c.

Fig. 54 is the shear diagram for the leeward post.

The moment diagram (Fig. 55) for the leeward post shows a zero moment at the point of contraflexure e', and a bending moment H_1k at the base, which must be cared for by the anchorage. Proper attention should be paid to the signs of the bending moments to note on which side of the post they cause compression or tension. It is also to be noted that the use of H_2 and H_3 is only a temporary device to show more clearly the action of the bending forces on the post.

CHAPTER II.

INFLUENCE LINES FOR SIMPLY SUPPORTED BRIDGE TRUSSES.

An influence line is a line showing the variation in any function at any section of a beam or in any member of a truss caused by any load moving along such a beam or truss. It is clear that such a line can be used to indicate the position of the moving load causing the *maximum* shears, moments, reactions, or stresses in any structure, and it is for the purpose of indicating and obtaining such maxima that influence lines are used.

In general, influence lines are drawn for a single unit load, and unless noted otherwise, it will be assumed that they are drawn for such loading only.

Art. 1.—Influence Line for Reaction.

A reaction influence line is a line showing the variation of the reaction of a beam under a moving load. If AB (Fig. 1) represents a simple non-continuous beam of length l, and P_1 a load moving over the beam from right to left, its distance at any instant from the right abutment being represented by x, the reaction R at the left will be given by eq. 1, which is the equation of a straight line:

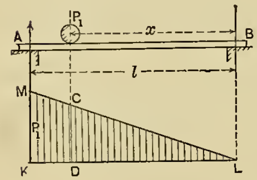


Fig. 1.

$$R = \frac{P_1 x}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If a line KL, parallel to the beam and of equal length, be laid off below the beam and an ordinate be erected at the left-hand end representing to scale the load P_1, a line, ML, connecting the end of this ordinate with the opposite end of the base line will be a line expressing eq. 1 graphically. If the load be at the distance x from B and an ordinate CD be erected immediately below this point, the triangles CDL and MKL will be similar, and therefore

$$\frac{CD}{DL}=\frac{MK}{KL}, \quad \text{or} \quad CD=\frac{P_1x}{l}=R.$$

This equation shows that an ordinate between the line ML and KL will represent the reaction at the left-hand end of the beam for the load P_1 placed at a point immediately over the ordinate; therefore the line ML is an influence line for reaction, since it shows the variation of the reaction as the load P_1 moves along the beam.

The influence line thus constructed for a load P_1 may be used for any other load Q, it being necessary, however, to multiply the value of any ordinate drawn for P_1 by the ratio of Q/P_1. This applies to any influence line which may hereafter be constructed.

Art. 2.—Influence Line for Shear.

An influence line representing the variation of shear at any section, CD, in a beam (Fig. 2) as a load crosses the span is derived in a similar manner. As a load P_1 advances towards the section from the right, the shear at any instant will be equal to the reaction R, or P_1x/l, and, as before, may be represented graphically by the line LM.

After passing the section, however, the shear becomes equal to the reaction R minus P_1, and is therefore a

negative quantity. To represent this graphically, ordinates of a value P_1 must be drawn downward from the line OM, and the locus of the ends of these ordinates will be the line KQ parallel to the line LOM. The line $LOQK$ will then represent the variation of the shear at the section CD as the load P_1 crosses the span, the shear being positive and of an increasing positive value as the load advances towards the section, and negative and of a decreasing negative value as the load leaves the section. It is seen that the maximum positive shear is found when the load is just to the right of the section in question. In practice the load is always placed at the section.

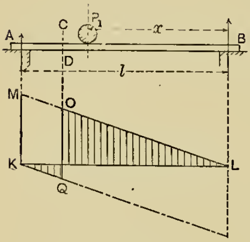


FIG. 2.

If two equal loads, P_1, separated by a fixed distance a, be employed, the maximum shear will be found when one of the loads is at the section, and the value of the shear will be the sum of the ordinates erected below the two wheel loads.

If two unequal loads, P_2 and P_3, separated by a fixed distance a, be employed, maximum values of the shear will again be found with one load at the section. If the influence line be drawn for a load P_1, then for the load P_2 placed at the section, both loads P_2 and P_3 being on the beam, the shear will be represented by the algebraic sum of the product of the ordinate corresponding to P_2 by the ratio of P_2 to P_1, and of the product of the ordinate corresponding to P_3 by the ratio of P_3 to P_1.

If the loading advances to the left to such a position that the load P_3 is at the section, the shear will be represented by the algebraic sum of corresponding products. Trial alone determines which position causes the greater maximum shear at the section; the second position of the loading will usually give the greater maximum shear, if

P_2 be small compared with P_3, if the distance a be large, or if the section CD be near the left abutment.

This construction can be used in finding the maximum shear at any section of a beam when a series of concentrated loads separated by fixed distances, as in the case of a locomotive, is used. The operation in such a case is as follows:

An influence line for a unit weight having been drawn, the algebraic sums of the products of ordinates erected under the various wheel loads by the actual weight of each load must be compared for positions of the loading with different wheel loads at the section in question; the values of the quantities so found will indicate not only the position of the loading for maximum shear, but will give the value of the shear. This operation is not as tedious as it may at first appear, since it is evident that the greatest maximum shear usually occurs with the head of the locomotive near the section. Attention is called to the fact that should the loading advance so much to the left that new loads appear upon the span at the right, such new loads must not be neglected.

Art. 3.—Influence Line for the Reactions of a Series of Concentrated Loads.

The influence lines so far considered have involved in their construction the use of only one load; and the use of such lines, in the case of more than one load, has required the use of arithmetical calculations. There will now be considered the construction of a line such that the variation of the reaction of a beam, as well as the reaction itself, when a series of concentrated wheel loads passes over the beam, may be measured directly from the drawing.

In order to simplify the explanation, only three loads,

P_1, P_2, and P_3, separated by the fixed distances a and b, will be used, but the construction is general and may be applied with ease to any number of loads. Let the positions of the loads be shown in Fig. 3, x representing the

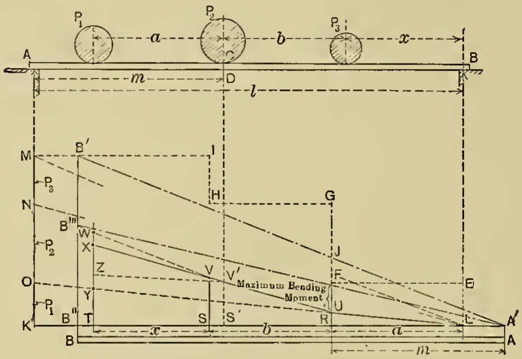


FIG. 3.

distance of P_3 from the right abutment. Taking moments about the right abutment, there will be obtained

$$R = \frac{1}{l}[P_1(a+b+x)+P_2(b+x)+P_3x]. \quad . \quad . \quad (1)$$

As before, let KL represent the base line equal in length to the span, and let KM represent to scale the loads P_1, P_2, P_3, laid off upwards from K in consecutive order. From L lay off to the left, in order and to proper scale, the distances a, b, and x. Draw the lines ML, NL, and OL. At R erect an ordinate RU to the line LO. From U draw a line UV parallel to NL to its intersection with an ordinate erected at S. From V draw a line VW parallel to LM to its intersection with an ordinate erected a T. *Then WT will represent the reaction at the left end of the beam for the positions of the loads as shown;* for WT is composed of the three parts, TY, YX, and XW, found

by continuing LU and UV to intersect WT. By construction, the triangles OKL and YTL are similar. Therefore

$$\frac{YT}{TL}=\frac{OK}{KL}.$$

Substituting the values of those quantities which are known, it is found that

$$YT=\frac{P_1(a+b+x)}{l}.$$

The triangles NOL and XYU are also similar, by construction; therefore

$$\frac{XY}{TR}=\frac{NO}{KL}.$$

Again, substituting the values of those quantities which are known,

$$XY=\frac{P_2(b+x)}{l}.$$

By construction, the triangles MNL and WXV are also similar. Therefore

$$\frac{WX}{TS}=\frac{MN}{KL}, \quad \text{or} \quad WX=\frac{P_3 \cdot x}{l}.$$

By summation,

$$YT+XY+WX=\frac{1}{l}[P_1(a+b+x)+P_2(b+x)+P_3x],$$

or the reaction at the left-hand end of the span, as already shown by eq. (1).

The loads may move any other distance x upon the bridge. This involves no change in the figure as drawn. To find the reaction for such a change in the position of the loading, an ordinate must be erected at a distance from the left end of b equal to the new distance x; *the intercept on this new ordinate between KL and the line VW continued will give the new reaction.* The line $LUVW$ is thus an influence line for reactions, since it shows the variation in the reaction as the loads P_1, P_2, and P_3 move along the bridge. Thus the ordinate immediately below P_1 always represents the reaction for that position of the loading.

In using a uniform load in this form of construction, the uniform load should be treated as a series of concentrated loads spaced as closely as the accuracy of the problem may demand.

Art. 4.—Influence Line for Maximum Shear, for a Series of Concentrated Loads.

It may now be shown that the line constructed in the manner described in the preceding article can also be used as an influence line to find the maximum shear at any section of a beam. Let CD, Fig. 3, be the section of the beam under consideration. If P_1, or the first wheel load, is at this section, the shear is equal to the reaction due to the loads on the beam or to $S'V'$. If the loading advances till P_2 is at the section, the shear at the section becomes equal to the reaction, represented by the ordinate WT minus P_1. P_1 can therefore be laid off downward from W as WZ. The shear is then represented by TZ. If the line $V'Z$ slopes downward to the left, it is evident that the shear with wheel load P_1 at the section is the greater; if the line slopes upward to the left, it is evident that the shear is greater with the wheel load P_2

at the section. Should P_2 give a greater shear, it is a simple matter to test wheel load P_3 at the section, or, in actual practice, any number of wheel concentrations, as in the case of a locomotive. The line $LUV'Z$ . . . is evidently an influence line for shear at the section CD.

Art. 5.—Influence Line for Moments.

A line which indicates the variation of bending moment at any point in a beam under a single moving load is a moment influence line. Let AB (Fig. 4) represent a beam

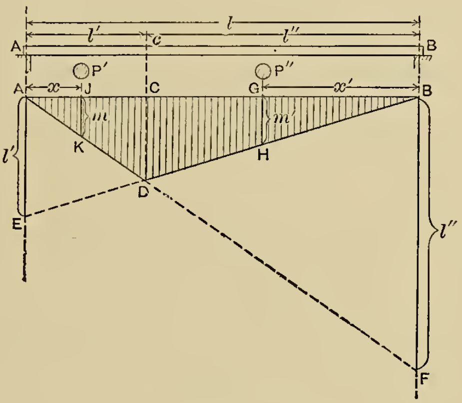


FIG. 4.

of length l, and let c be the section which divides the beam into the two portions l' and l'' and about which moments are to be found. If a load P'' be on the section l'' (at the point G, distant x' from B, for instance), the moment M' about C is the product of the left reaction by the length AC; that is,

$$M' = \frac{P''x'}{l} \cdot l' = P''m'. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the load P'' is a unit load, eq. (1) takes the form

$$M' = \frac{x'}{l} \cdot l'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eq. (2) is the equation of a straight line and may be represented graphically as follows: Erect on the base line AB at A a vertical line AE equal in length to l', and connect E with B, the opposite end of the span. From similar triangles $GH : l' :: x' : l$; that is,

$$GH = \frac{x'}{l} \cdot l'.$$

Therefore any ordinate between CB and DB represents the bending moment at C when the unit load is placed directly over such ordinate. Similarly, for a unit load on l' between A and C and measuring x from A, the moment is

$$M'' = \frac{xl''}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the line AD, representing eq. (3) graphically, is drawn by connecting F, the end of a vertical l'' erected at B, with A. The line ADB is thus an influence line for moments. It is evident from the construction that the corner D must lie vertically below the centre of moments.

If P' is the load on l', the general value of the moment M'' represented by eq. (3) is

$$M'' = \frac{P'x}{l} l'' = P'm. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

If the load is uniform over the entire span and of the intensity p, i.e. p being the amount per linear unit, P'' becomes $p \cdot dx'$ and P', $p \cdot dx$. By substituting these values

in eqs. (1) and (3a) and remembering that differential moments then result,

$$dM' = p\frac{x' \cdot dx'}{l}l' \quad \text{and} \quad dM'' = p\frac{x \cdot dx}{l}l''.$$

The moment M at C then takes the value

$$M = \frac{p}{l}\left(\int_0^{l''} x' \cdot dx' \cdot l' + \int_0^{l'} x \cdot dx \cdot l''\right) = \tfrac{1}{2}pl'l''. \quad . \quad (4)$$

Obviously M has its greatest value at the centre of span where $l' = l'' = \frac{l}{2}$ and for which

$$M = \frac{pl^2}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It is also clear from the preceding that the area of the triangle ABD multiplied by p represents the value of the moment M at C for a uniform load over the entire bridge; its value is

$$M = \tfrac{1}{2}AB \cdot CD \cdot p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But $CD:AE::BC:AB$, therefore

$$CD = \frac{AE \cdot BC}{AB} = \frac{l'l''}{l}.$$

Substituting this value in eq. (6), $M = \frac{1}{2}pl'l''$, as in eq. (4).

Art. 6.—Criterion for Maximum Moment at any Section of a Beam.

By means of the influence line of the preceding article, the criterion for the position of a series of concentrated loads producing the maximum moment at any section of a beam may be deduced. Let P'' be any load on l'' (Fig. 4), and m' the value of the influence ordinate corresponding to this load; P' any load on l', and m the corresponding influence ordinate; x' the distance of P'' from B, and x the distance of P' from A; and then let M be the value of the moment at C for any position of the wheel loads. If the sign Σ indicates the summation of terms of the same kind,

$$M = \Sigma(P'm) + \Sigma(P''m'). \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The values of m and m' may be expressed as follows:

$$m = \frac{CD \cdot AJ}{AC} = \frac{CD \cdot x}{l'},$$

$$m' = \frac{CD \cdot BG}{BC} = \frac{CD \cdot x'}{l''};$$

$$\therefore \; M = CD\left[\Sigma\left(\frac{P' \cdot x}{l'}\right) + \Sigma\left(\frac{P''x'}{l''}\right)\right]. \quad . \quad . \quad . \quad (2)$$

If the loads advance a distance Δx to the left, the value of M becomes

$$M + \Delta M = CD\left\{\Sigma\left[\frac{P'(x - \Delta x)}{l'}\right] + \Sigma\left[\frac{P''(x' + \Delta x)}{l''}\right]\right\}. \quad . \quad (3)$$

The change in the value of M is therefore

$$\Delta M = CD\left\{-\Sigma\left(\frac{P'\cdot\Delta x}{l'}\right)+\Sigma\left(\frac{P''\cdot\Delta x}{l''}\right)\right\}. \quad . \quad . \quad (4)$$

For a maximum or minimum $\Delta M = 0$, hence

$$\frac{\Sigma P'}{l'}=\frac{\Sigma P''}{l''}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If ΣP represents $\Sigma P' + \Sigma P''$, then eq. (5) may take the form

$$\frac{\Sigma P}{\Sigma P'}=\frac{l}{l'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Eq. (6) is the criterion desired; it represents an equation whose conditions must be fulfilled for maximum moment.

Art. 7.—Maximum Moments in a Beam.

It should be observed that the influence line for reactions, as found in Art. 3, is a funicular polygon for which the pole distance is the perpendicular distance between the pole L and the line MK. This polygon differs from the ordinary funicular polygon, however, in that the various loads and distances are laid off in order exactly reverse to the usual procedure; that is, the loads are laid off upwards beginning with P_1, and the distances a, b, etc., are laid off from right to left, beginning at the right with a. This brings the head of the moving load to the right, whereas in the usual procedures the head of the moving load is at the left. The significance of this construction should be carefully noted.

In finding the maximum moments in a beam, it will be necessary to use the funicular polygon as a moment polygon in the case of parallel forces and also to use the criterion deduced in the preceding article.

In order that the condition of equilibrium expressed by that equation may hold, it will usually be found necessary to place a load directly at the section, since any portion of this load may be considered to be on either side of the section. This criterion is easily adapted to graphic construction.

In Fig. 3 of Art. 3 draw LE equal to P_1 vertically above L; from E draw a horizontal line until it meets at F an ordinate drawn vertically from R. Lay off FG equal to P_2; from G draw a horizontal line GH, etc.; follow this same form of construction for the remainder of the oads. The result will be the stepped diagram $LEFGH$ . . . representing graphically the summation of all the loads, and also of all the distances from the head of the moving load to any point in the moving system.

In the case in hand it is desired to find the position of the moving load causing the maximum bending moment at any section CD distant m from the left abutment. Lay off to scale on the edge of a strip of paper the length of the beam l, and from the *right-hand* end of l lay off m. It is important to notice that although the section is distant m from the left abutment, the distance m must be measured from the right-hand side on the funicular diagram, since this polygon is in a reverse position to the usual. Move the strip of paper until the load P_2 or FG is over the section under consideration. If this position gives a maximum bending moment, the criterion must be satisfied. Erect vertical ordinates at the ends of the beam and at the section until these ordinates intersect the stepped diagram at A', B', and J. Draw a line from A' to B'; if the line $A'B'$ passes through the step of the diagram

representing the load situated at the section, the criterion will be fulfilled and this position of the loading will cause maximum bending. For, in the similar triangles RJA' and $B''B'A'$,

$$\frac{RJ}{B''B'}=\frac{RA'}{A'B''}; \text{ that is, } \frac{\Sigma P'}{\Sigma P}=\frac{l'}{l}.$$

Should the line $A'B'$ not pass through the step GF, the strip of paper must be moved until another load is brought to the section and the construction above described must be repeated.

If, as in the present case, the criterion is satisfied, the bending moment can be immediately obtained. Erect vertical ordinates at the ends of the beam until they intersect the influence line at B''' and A'. Connect these points of intersection. As in Chap. I, Art. 10, the value of the vertical intercept between this line $B'''A'$ and the influence line, when multiplied by the pole distance, will give the bending moment at any section. In this way the one diagram can be made to serve in finding both maximum shears and maximum bending moments in a beam.

In actual practice it may be found advisable in constructing the influence line to take as a pole distance not the entire length of the beam, but only a fractional part of it. This will cause the influence line to have a steeper inclination and will tend to make more accurate the measurement of all vertical heights, which evidently will be all increased in the same ratio. Care must be taken in such a case to give to all ordinates, whether for shear or bending moment, their proper values.

Art. 8.—Maximum Stresses in the Web Members of a Truss with Parallel and Horizontal Chords.

The graphical operations of the preceding articles are sufficient to furnish all the maximum stresses in trusses with parallel and horizontal chords. In finding the maximum web stresses use will be made of the following principle: The stress in any web member of a truss having parallel chords is equal to the shear in the panel multiplied by the secant of the angle which the member makes with a vertical. If the shear is known, the stress can be found very simply by a graphical construction. Let the vertical side of a right-angled triangle represent the shear to scale, and let the hypothenuse make an angle with this line equal to the inclination of the web member with a vertical. Then the hypothenuse will represent the stress in the member to scale.

Therefore, in order to find the maximum stress in a web member, it will be necessary to find the maximum shear in the panel in which the member is situated, and this shear will be found graphically by means of an influence line differing but slightly from the influence line for maximum shears in a beam. The change involved arises from the fact that the loading on a truss is applied at the panel points of the chords. It is clear that this application of the loads affects only the panel under consideration, for in the other panels the loads, whether considered as they stand or as if concentrated at the panel points, cannot influence the shear in the panel under consideration.

The shear in any panel is constant between panel points, and is equal to the reaction at the left end of the truss minus all the loads which may be situated in the panels to the left of the panel under consideration, and

minus that portion of the loading situated in the panel itself which is transferred to its left end. It is evident that for maximum shear no load should pass the panel itself. There only remains to be considered the distance which the locomotive must advance on the panel in question.

As an illustration, the maximum stresses in some of the web members of an 8-panel, 208-foot Pratt truss will be determined, using as the loading the locomotive concentrations designated in Cooper's specifications as *E* 40, and assumed to advance from right to left.

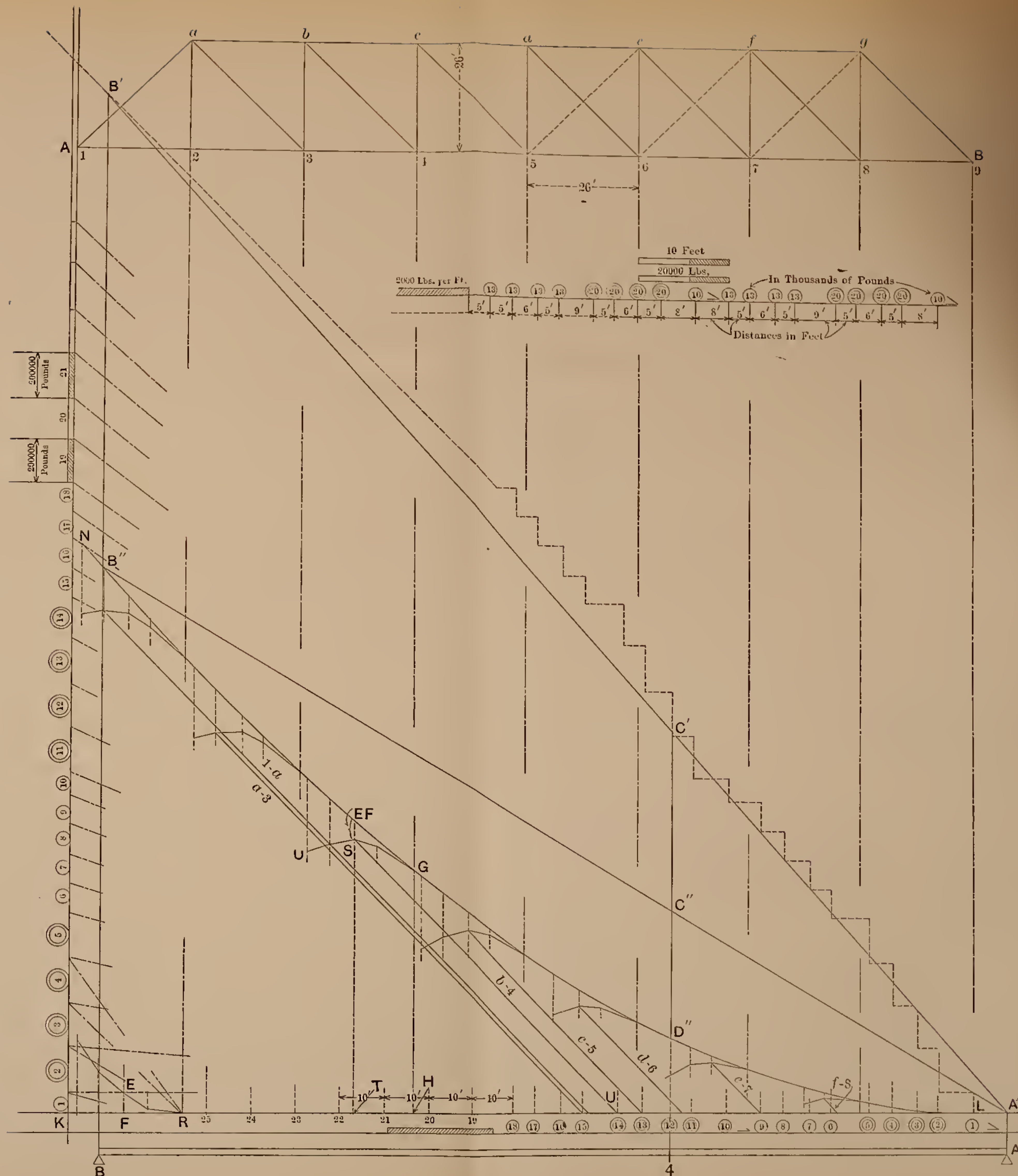
Let *AB*, Fig. 5, represent the truss under consideration. Draw the base line *KL* equal in length to the total length of the truss. Beginning at *L* and with wheel 1 lay off on *KL* and towards the left, to scale, the distances between the various wheel loads, and erect vertical ordinates at the points thus found. It should be noted that the uniform load is treated as a series of concentrated loads, each concentration representing ten feet of uniform load. At *K* erect the vertical *KM*, and lay off on *KM* upwards, to scale, the amounts of the loads, beginning at *K* with wheel 1. Connect the points thus found with *L*, which becomes the pole of the funicular polygon, and by the aid of these lines draw the funicular polygon $LD'' \ldots N$. As already demonstrated, this is an influence line for reactions and for shears for a beam with the span *KL*.

In order to use this line in the present case, it will be necessary to find the reactions at the left-hand end of any panel when loads are found within such panel. For this purpose, a similar influence line must be drawn, using in this case, however, the panel length as the length of span. This second influence line is shown in the lower left-hand corner of the figure as *RE*; in this case *R* is the pole.

In order to find the maximum stress in any member, such as b-4, for example, it will be necessary to find the maximum shear in the panel 3-4: the two influence lines already constructed are then to be combined for this purpose. From panel point 4 let fall a vertical ordinate upon the line $LD'' \ldots N$, cutting it at G. GH represents the shear at the panel point 4, and also in the panel 3-4 when wheel load 1 is at this point. If the load advances so that wheel load 2 is at this panel point, the shear cannot be measured directly from the original influence line $LD'' \ldots N$ with its value diminished by wheel load 1; the shear is now diminished only by that portion of wheel load 1 which is carried to panel point 3. The value of this portion is known from the line RE, and its position relative to the panel point 4 is also known. Therefore this value is drawn at the proper point as a negative ordinate from the line $LD'' \ldots N$.

The same construction is followed for other positions of the loading, in which other drivers are at panel point 4. The final influence line is GSU, and ordinates between the base line KL and GSU will then show the shear in the panel, as the loading advances toward the left. By inspection it is then seen that the maximum shear is TS; by employing the construction noted in the beginning of this article, the line SU', drawn parallel to b-4, will represent to scale the stress in b-4. The stress in the vertical b-3 is found at the same time and is equal to the maximum shear in the panel or TS.

A similar method of procedure is followed in the case of all the other web members, including as such also the end post. The graphical operations may be performed very expeditiously by treating the same load at one time in all the panels. By this means the counter-stresses in the members are also found at once, for the stresses in those members situated to the right of the centre of the



10 Feet
20000 Lbs.
2000 Lbs. per Ft.
In Thousands of Pounds
Distances in Feet
200000 Pounds
1-a
a-3
b-4
c-5
d-6
e-7
f-8

truss indicate the maximum compression which can exist in the corresponding members to the left of the centre; For convenience the stresses in these members have been found as if they sloped downward to the right instead of downward to the left.

The dead-load stresses can be found in the usual manner by a force polygon, or by representing the shears in the truss by a stepped diagram, and using the construction employed above. By comparison with the live-load stresses, the necessity of counter-braces is at once determined, and also the values of their stresses.

Art. 9.—Maximum Stresses in the Chord Members of a Truss with Parallel Chords.

In order to find the maximum stresses in the chord members of a truss with parallel chords, the principle of sections will be employed. The method used, however, can only be applied, as far as the present analysis is concerned, to those trusses in which the centre of moments is found in a vertical line drawn through a panel point of the loaded chord. The truss and loading represented in Fig. 4 will again be employed and the maximum stress in the upper chord member b–c will be determined. Panel point 4 is the centre of moments for this case; therefore the maximum moment at the point 4 must be found. As already noted, the funicular polygon $LD'' \ldots N$ is in a reversed position to the loading as it advances on the truss from right to left. Let the line BA below the line KL represent the truss laid off to scale on a strip of paper, the point 4 representing the centre of moments. By use of the stepped diagram, explained in Art. 7, it is found that wheel 9 at the section gives one position for maximum bending. By erecting ordinates AA' and BB'' and drawing the line $A'B''$, the bending moment $C''D''$ is graphically

obtained, but it must be multiplied by the pole distance in order to be expressed in proper units. The maximum live-load stress in b-c is the bending moment thus found divided by the depth of truss. The dead-load stress is found either by a force polygon or by means of a moment curve.

Art. 10.—Influence Lines between Adjacent Panel Points.

Beams and girders as a rule carry the loads imposed upon them at any point in their spans. In the usual types of trusses, however, and sometimes even in the case of long girders, the load is brought upon the structure at fixed panel points only. Due to this redistribution of the load at panel points, it becomes necessary to investigate the change occurring in influence lines which may have been drawn under the assumption that a load acts where it appears to be placed. Let Fig. 6 represent two adjacent panel points designated as points $n-1$ and n, and let the load P be distant x from n. The effect of the load P on the members of the truss will be that due to its distribution to the adjacent panel points. In this case the equivalent effect is that due to forces equal in amount but opposite in direction to the reactions of a load P on a span of panel length l. If these forces are represented by R_{n-1} and R, there will be found

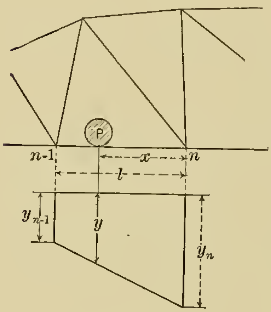


FIG. 6.

$$R_{n-1}=\frac{Px}{l}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$R=\frac{P(l-x)}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let the ordinates of the influence line beneath the panel joints and beneath the load be designated respectively by y_{n-1}, y_n, and y. Since the product of the load P by its influence ordinate must be equal to the sum of the products of its parts, each multiplied by its corresponding coordinate, there will be obtained

$$Py=R_{n-1}\cdot y_{n-1}+R_n\cdot y_n. \qquad (3)$$

By the substitution of the values of eq. (1) and eq. (2) in eq. (3) there will be found

$$y=\frac{x}{l}y_{n-1}+\frac{l-x}{l}y_n. \qquad (4)$$

In this expression x is the variable quantity, and since it appears only in the first degree, the line which eq. (4) represents must be a straight line. The following rule may therefore be deduced: An influence line for a *single* moving load showing the variation in any function for any part of a truss is a straight line between adjacent panel points.

Art. 11.—Moment Influence Lines for Any Truss.

If the centre of moments lies on a vertical which intersects the loaded chord line within the limits of a panel, the moment influence line treated in Art. 5 must be modified in consequence of the redistribution of the loads within the panel to the panel ends.

In finding the stress in the chord member BD, Fig. 7, where the centre of moments C falls within the panel BD, the influence line drawn without reference to the existence of the panel is only correct for the portions of the truss AB and DE, since in finding the moment at C the loads

on these portions of the truss are not redistributed. It has been shown, however, (Art. 10) that all influence lines for single loads between panel points are straight lines;

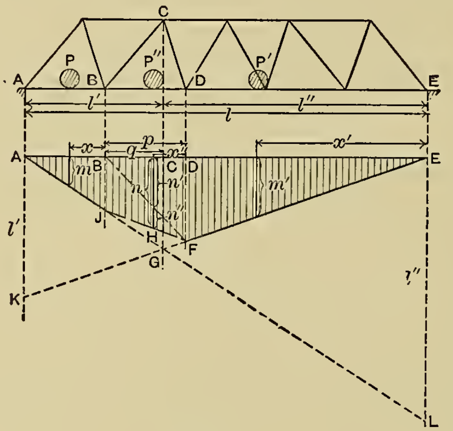


FIG. 7.

therefore that portion of the influence line which lies within the panel is then constructed by connecting J and F, the points of intersection of the influence line already drawn with the verticals erected at the panel ends. The influence line is then $EFJA$.

In order to find the position of a series of concentrated loads causing the maximum moment at the section C, it will manifestly be incorrect to use the criterion found in Art. 6, but the proper criterion is easily deduced.

The following notation (Fig. 7) will be employed:

l' and l'' = the distances from the left and right abutments to the centre of moments respectively;
l = the length of span;
p = the length of panel;
q = the distance from the centre of moments to the left end of the panel;

P and P' = any loads on the sections AB and DE respectively;
P'' = any load within the panel BD;
x, x', and x'' = the respective distances of P, P', and P'' from the right ends of the various sections;
m, m', and n = the respective influence ordinates for P, P', and P''.

Then, as usual, the moment M for any position of the loads becomes

$$M = \Sigma(P \cdot m) + \Sigma(P'' \cdot n) + \Sigma(P' \cdot m'). \quad . \quad . \quad (1)$$

If the ordinate n is divided into two parts n' and n'' by the line BF, then the moment M for any position of the loads, using a notation similar to that before, becomes

$$M = \Sigma(P \cdot m) + \Sigma(P'' \cdot n') + \Sigma(P'' \cdot n'') + \Sigma(P' \cdot m'). \quad (2)$$

Eq. (1) may take the following form by substituting for the influence ordinates equivalent values found from the similar trangles of Fig. 7:

$$M = \Sigma\left[P \cdot BJ\frac{(AB - x)}{AB}\right] + \Sigma\left[P'' \cdot \frac{DF(BD - x'')}{BD}\right]$$
$$+ \Sigma\left[P'' \cdot \frac{BJ \cdot x''}{BD}\right] + \Sigma\left[P' \cdot \frac{DF \cdot x'}{ED}\right]. \quad (3)$$

If the loads advance a distance Δx to the right, the change in the values of the moment becomes ΔM and is equal to

$$\varDelta M = \Sigma\left[P\cdot\frac{BJ\cdot\varDelta x}{AB}\right] + \Sigma\left[P''\frac{DF\cdot\varDelta x}{BD}\right] - \Sigma\left[P''\cdot\frac{BJ\cdot\varDelta x}{BD}\right]$$

$$- \Sigma\left[P'\cdot\frac{DF\cdot\varDelta x}{ED}\right]$$

$$= \Sigma\left[P\cdot\frac{l''}{l}\cdot\varDelta x\right] + \Sigma\left[\frac{P''}{p}\cdot\frac{l'(l''+q-p)\varDelta x}{l}\right]$$

$$- \Sigma\left[\frac{P''}{p}\cdot\frac{l''(l'-q)\varDelta x}{l}\right] - \Sigma\left[P'\cdot\frac{l'}{l}\cdot\varDelta x\right]$$

$$= \Sigma\left[P\cdot\frac{l''}{l}\varDelta x\right] + \Sigma\left[\frac{P''}{pl}(ql-l'p)\varDelta x\right] - \Sigma\left[P'\frac{l'}{l}\varDelta x\right].$$

For a maximum or minimum $\varDelta M = 0$; noting that

$$\Sigma\left[P\frac{l''}{l}\right] = \Sigma\left[P\cdot\frac{l}{l}\right] - \Sigma\left[P\frac{l'}{l}\right],$$

there is found

$$\Sigma P - \Sigma\left[(P+P'+P'')\frac{l'}{l}\right] + \Sigma\left[P''\frac{q}{p}\right] = 0. \quad . \quad . \quad (4)$$

If ΣW represent all the loading on the span, then eq. (4) will take the form

$$\Sigma P + \frac{q}{p}\Sigma P'' = \frac{l'}{l}\Sigma W. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This criterion is used in the same manner as the criterion deduced for finding maximum moments in a simple beam, but it differs from that in one respect, viz., the left-hand member of eq. (4) does not include the sum of all the weights in the panel, but only a definite portion. It

will usually be found that the conditions of the equation are fulfilled by placing a wheel load at the left end of the panel.

Art. 12.—Variation of Moment within a Panel for a Fixed Position of the Loading.

Before proceeding with an example showing the appli-action of the criterion just deduced, it is advisable to show that if a series of concentrated loads is fixed in position on a truss, the variation of the bending moment between panel points may always be represented by a straight line. Let it be required to find the variation of bending moment in the panel p of the truss shown in Fig. 8, the position of

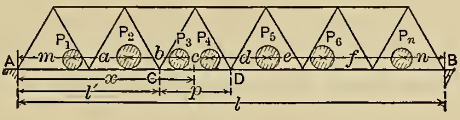


FIG. 8.

the wheel loads being fixed. Let R be the reaction at A, the left end of the truss, for all the loads shown on the truss, and R_1 the reaction at the left end, C, of the panel CD, for the loads within the panel; the other notation to be employed is shown in the figure. Then

$$R = P_1[(a+b+c+\ldots+n) + P_2(b+c+d+\ldots+n)\ldots + P_n \cdot n \ldots]\frac{1}{l}.$$

$R_1 = [P_3(c+n') + P_4(n') \ldots]\frac{1}{p}$, n' being the distance between P_4 and the right end D of the panel.

If M represents the moment at any point in the panel p distant x from A,

$$M = R \cdot x - P_1(x-m) - P_2(x-a-m) \ldots - R_1(x-l'). \quad (1)$$

Eq. (1) shows that between panel points M varies directly as the first power of x, and that the variation may consequently be represented by a straight line. Therefore to find the moment at any section within a panel, for a fixed position of the loading, it suffices to know the moments at the panel ends; by connecting the ends of the ordinates representing these end moments by a straight line, any intermediate moment is at once obtained. This method is easily applicable with a funicular polygon, since the latter is a moment polygon for parallel loads.

Art. 13.—Problem in Finding the Maximum Stress in the Loaded Chord Member of a Truss with Web Members all Inclined.

An application of the principles of the preceding articles will be made in finding the maximum stress in the loaded chord member 3-4 of the 6-panel deck-truss shown in Fig. 9. The following data are given:

Panel lengths, all equal............	29	feet
Length of truss....................	174	"
Depth of truss at panel point a....	19	"
Depth of truss at panel point b.....	24	"
Depth of truss at panel point c.....	25	"

Locomotive loading: E 40. Cooper's Specifications.

The reaction influence line $LMNPQ$ . . . and the stepped diagram $LABCDE$ . . . are drawn in the usual manner. The truss itself is redrawn on a separate strip of paper, but in the reverse position to that of the upper diagram, the reason for which has already been explained (Art. 7). It is shown in dotted lines in the lower part of Fig. 9. This strip of paper must then be placed in such a position that the criterion of Art. 11,

$$\Sigma P + \frac{q}{p}\Sigma P'' = \frac{l'}{l}\Sigma W,$$

is satisfied with respect to the member 3-4.

In this case, q is the distance 3 to c'; p is the distance 3 to 4; l' is the distance 1 to c'; l is the distance 1 to 7.

Wheel 8 will rest at panel point 3 if the truss be placed in the position shown in the lower part of the figure. Verti-

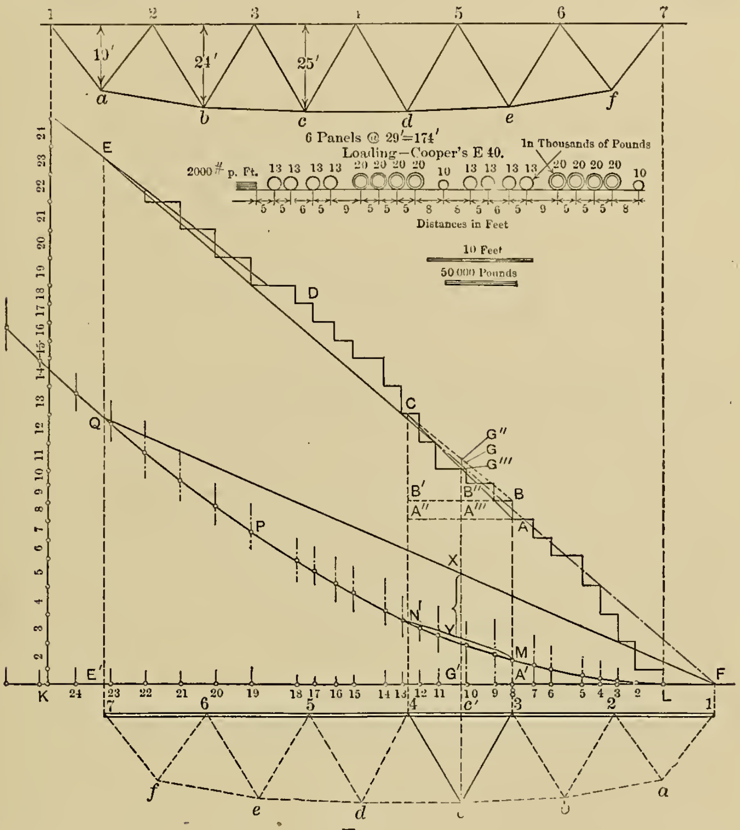


FIG. 9.

cals erected at the ends of the truss and at the panel points 3 and 4 will intersect the stepped diagram at the points F E, A, and C respectively.

ΣW is then represented graphically by EE', and ΣP either by $A'A$ or $A'B$, since wheel 8 may be taken to act

either to the right or left of the panel point. For a similar reason, $\Sigma P''$ may be represented correspondingly either by $A''C$ or $B'C$. From similar triangles,

$$GG' : EE' :: FG' : FE';$$

that is,

$$GG' = \frac{l'}{l}\Sigma W.$$

According to the criterion, GG' must equal either

$$A'A + \frac{q}{p}A''C$$

or

$$A'B + \frac{q}{p}B'C.$$

From similar triangles it is seen that

$$\frac{q}{p}A''C = A'''G''', \quad \text{and that} \quad \frac{q}{p}B'C = B''G''.$$

Therefore, by substituting these values in the criterion, the value of GG' should equal either $G'G'''$ or $G'G''$. In the present case $G'G$ lies between these two values; it is evident that if wheel 8 be divided into proper parts at the panel point, the conditions of the criterion will be exactly fulfilled. In practice, therefore, it is only necessary to determine whether the point G lies between the lines BC and AC; if it does, as in the present case, this position of the loading will furnish one maximum value for the stress in 3-4.

It will usually be found that more than one position of the loading will fulfil the required conditions; in such a case the actual values of the stresses must be determined and the absolute maximum taken.

The value of the moment at C is found by means of the method of Art. 12. The closing line FQ of the funicular polygon is drawn and the points M and N, corresponding to the panel points 3 and 4, are connected by a straight line; the intercept XY when multiplied by the proper pole distance will give the value of the moment; in this case 8,734,000 foot-pounds. The stress in 3-4 is this moment divided by the lever-arm cc'; that is, $\frac{8734000}{25} =$ 349,000 pounds compression. The stresses in the other members of the loaded chord will then be found in a precisely similar manner.

Art. 14.—Determination of Stresses in Three Non-concurrent Members of a Truss.

A frequent problem* in statics is the determination of the stresses in three truss members not meeting in a point. It is assumed that the external forces are fully known in regard to the magnitude, direction, and point of application. In the following treatment these external forces may therefore be replaced by their completely known resultant. Let the section mn, Fig. 10, cut the

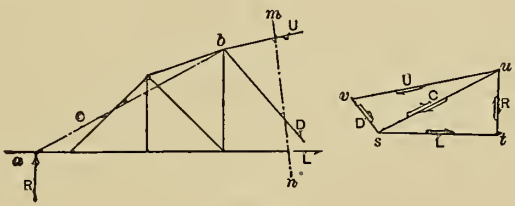


FIG. 10.

three members U, L, and D of the truss, and let R represent the position, direction, and magnitude of the resultant of the external forces situated at the left of the

* See Chap. I., Art. 12, for a solution of the same problem, but stated in more general terms.

section and which holds in equilibrium the stresses in the members in question.

Continue L till it intersects R at a. Then connect b, which is the intersection of U and D, and a by the line C, and which, for the present, may be assumed the line of action of a supplementary force acting through a and b. The force polygon *stu* may then be drawn by using the three concurrent forces R, C, and L, since only the amounts of C and L are unknown. This operation can be repeated with the forces C, U, and D, assuming in this instance C to be completely known. The resulting triangle is *suv*.

If C in the second case be given a direction opposite to that in the first case, it is found that the supplementary force C is annulled; in other words, the resultant of two of the forces balances the resultant of the other two. Thus the forces R, L, D, and U form a closed polygon, and the forces which are represented by the sides of this polygon are in equilibrium. The stresses D, L, and U are then completely determined.

Art. 15.—Stresses in the Web Members of any Simply Supported Truss.

In order to determine the variation of the stress in the web member or diagonal D of any truss such as shown in Fig. 11, in which the moving load traverses the lower chord, let a load P move over the truss from the right abutment to panel point 3. By the method of sections and moments it can be seen that the stress in D in this case is influenced simply by the reaction R at the left abutment; it is clear, therefore, that with the load P between panel points 3 and 5 the stress in D will always be of the same sign. In the same way for a load P between the left abutment and panel point 2 the stress in D is influenced only by the reaction R'. This stress will always be of an opposite

kind to that with the load between the points 3 and 5. It should, however, be noted that these statements do not apply to trusses of that kind in which the two chord members which are cut by the section meet within the limits of the truss.* In the following treatment, therefore, trusses of this character are excluded.

It must be evident that some point in the panel 2-3 (Fig. 11) must be a critical point for deciding how far

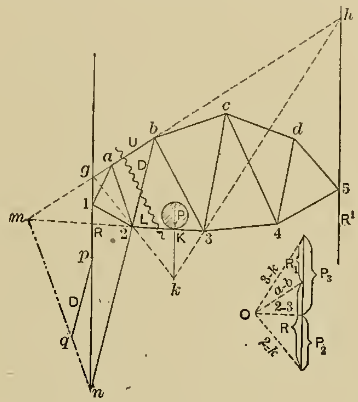


FIG. 11.

a load may advance into the panel before the stress in the diagonal of that panel is reversed. This critical point will be found where the load if applied will cause no stress at all in the member. The following construction will determine its location.

Continue the unloaded chord member *a-b* until it meets the lines of action of the reactions at *g* and *h*; then draw the lines *gk* and *hk* through the panel points 2 and 3 to the intersection *k*. A vertical line through *k* will determine the desired critical point, as will be shown. Assume any load *P* placed vertically over *k*. Its effect on the stresses in the members of the truss is the same as if it

* See Art. 22 of this Chapter.

were replaced by its components P_2 and P_3 acting at panel points 2 and 3, the amounts of which are found by means of the triangle $2k3$, a funicular polygon for this loading on a span equal to the panel length. The force polygon corresponding to this funicular polygon is shown in Fig. 11, and represents P_2 and P_3 to scale. In a similar manner $g23h$ is a funicular polygon for the forces R, R', P_2, and P_3, and the values of the reactions R and R' of the main truss are found on the same force polygon, Fig. 11, by drawing a line through the pole O of the force polygon parallel to ab. The forces whose moment is to be taken and which cause stress in D are therefore R and P_2. Their resultant, whatever be its value, acts at the intersection of the strings ab and 23 of the funicular polygon. But this point of intersection is also the intersection of the two chord members U and L, and is the centre of moments for finding the stress in D. The resulting moment of R and P_2, therefore, is zero and the diagonal sustains no stress for this position of the loading. Therefore loads advancing from R' to the critical point K will cause stress of one kind, and loads from R to K stress of the opposite kind. Trusses with the loading on the upper chord are treated in a precisely similar manner, the construction to be followed in any case being simply this: Continue the unloaded chord member cut by the section till it intersects the lines of action of the reactions; then draw lines from these points through the ends of the loaded chord member which is cut by the section, and the intersection of these lines will give the desired point.

If the moving load is a uniform load, its position for the maximum stresses of opposite kinds is immediately found; the loading extends alternately from either point of support to the critical section. The value of the stress itself may be found quickly in a manner which, although approximate, is sufficiently exact to cover the usual cases

occurring in practice. The stresses so found will have a value slightly greater than those which actually exist. Assuming the load to advance from the right abutment, the approximation consists simply in neglecting that portion of the loading acting downward at panel point 2, and considering the stress in the diagonal to be caused only by the reaction R. The value of R may be found very simply either analytically or graphically. By means of the construction explained in Art. 14 the stress in the member may then be found as follows:

The reaction R acting at 1 holds in equilibrium the three unknown forces U, D, and L. Continuing U and L to intersect at m, and letting D intersect R at n, the force acting along the line mn is found to balance these two pairs of concurrent forces. In order to obtain the value of D, lay off upwards from n the value of R equal to np. Through p draw a line parallel to D to intersect mn at q; pq will then be the stress in D, and will be completely determined in regard to amount and direction. In order to find the greatest counter-stress, the load must cover that portion of the truss not loaded before; otherwise the operation is precisely similar to that just described.

Art. 16.—Influence Line for Stress in any Web Member of a Simply Supported Truss.*

METHOD I.

As a unit load passes over the bridge from right to left, an influence line for the stress in any diagonal, such as AC of the truss shown in Fig. 12, may easily be drawn. In this case the load is carried at the panel points of the lower chord, but the methods to be used are also applicable to trusses in which the load is carried by the upper chord. It

* That type of truss, however, in which the chord members intersect within the limits of the span is again excluded (see Art. 15).

is clear that all loads between points R' and B will cause stresses in the diagonal of the same sign, and their aggregate magnitude will vary directly as the left-hand reaction. The variation of this reaction can be indicated by an influence line for reactions, and is a straight line for a unit load. Therefore the influence line for stress in the diagonal AC for a single load may be drawn upon the reference line

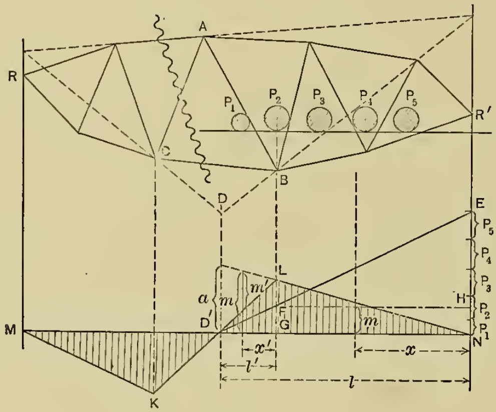


FIG. 12.

MN, as a straight line NL between R' and B; its slope must be fixed by locating two of its points.

Similar reasoning holds for the distance RC. Hence the influence line for this portion of the truss is also a straight line, but since the stress is of an opposite kind from that when the load is between R' and B, the line MK is drawn below the reference line. There still remains to be determined the influence line when the load is in the panel itself.

It has already been shown (Art. 15) that there is a point D in the panel where a load, if placed, produces no stress in the diagonal of that panel; and it has been shown (Art. 10) that all influence lines are straight lines between

panel points. Therefore the influence line in the panel itself is found by drawing any straight line through D' found vertically below D. The line $KD'L$ intersects verticals drawn through the panel points K and B. The influence line is then completed by connecting the points K and L with the ends of the reference line M and N, since it has just been shown that these portions are straight lines, and two points upon each of these lines have now been found. Ordinates above the reference line indicate stress of one kind, and ordinates below, stress of the opposite kind. The scale with which to measure the influence ordinates remains to be determined; but it is clear that if the stress is known for one position of the load, such as B, the scale is immediately fixed.

The influence line for a unit load having thus been found, the position of a series of concentrations causing the greatest stress can be found precisely as in Art. 2. The loads having been placed in a trial position, the sum of the products of the ordinates of the influence line by the amount of the load placed over any ordinate may be found and compared to another trial position. The greatest sum will give the position of the loading for maximum stress and, by means of the proper scale, the stress itself. It is possible, however, by the aid of the influence line just developed, to deduce a criterion which will at once enable the position of the loads causing the maximum stress to be determined. (Art. 17.)

The ease with which uniform loads may be treated is at once evident from Fig. 12; for maximum stress it is only necessary to cover the portion ND' of the truss; knowing the scale of the diagram, the stress in AC is the area $D'LN$ $=\frac{1}{2}D'N \times GL$. These distances are easily measured from a carefully executed diagram.

The scale of a stress influence diagram may be found as follows: Let there be placed at any panel point of the

loaded chord a unit load, and let there be drawn for this one load an ordinary stress diagram for the structure furnishing the stress for each member for the one load employed. These stresses provide at once a means of determining the scale of all influence-line diagrams, for they furnish for each influence line the value of the ordinate below the point of application of the load.

The use of influence areas becomes immediately available in actual design work, if an equivalent uniform load could be found to replace locomotive concentrations. Such an equivalent load, however, would vary not only with the span lengths, but also with the purpose for which it is to be used, that is, for moments or shears. It is becoming more the custom, however, in bridge-design offices to obtain an equivalent uniform loading for every span length, and for all conditions. Once obtained and tabulated, they may be quickly applied in all influence-line work.

METHOD II.

The variation of the stress of a web member, such as CG, in the truss shown in Fig. 13, may under certain conditions be more conveniently represented by the aid of the following analysis:

l_1 = that portion of the span length between the left abutment and the left end of the panel cut by the section;
l_2 = that portion of the span length between the right abutment and the right end of the panel cut by the section;
p = the panel length;
l = the length of truss;
m = the distance between the centre of moments and the left abutment;
x = the distance of any load from E, the centre of moments;

R_A and R_B = the left and right reactions respectively;
d = the lever-arm of CG about E (not shown);
S_A and S_B = the stress in the member for a single load on each span section l_1 and l_2 respectively.

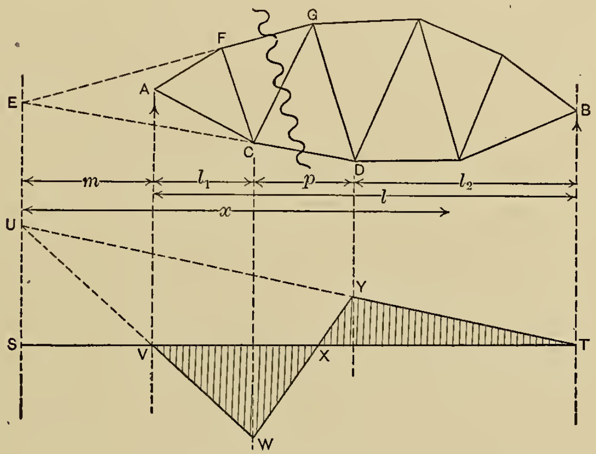


FIG. 13.

Then, for a unit load on l_1, the general expression for S_A becomes

$$S_A = \frac{R_B(l+m)}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But $R_B = \frac{x-m}{l}$;

$$\therefore S_A = \frac{(x-m)(l+m)}{l \cdot d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Similarly, for a unit load on the span l_2, the general value of S_B becomes

$$S_B = \frac{R_A \cdot m}{d}.$$

But $R_A = \frac{l+m-x}{l}$;

$$\therefore S_B = \frac{(l+m-x)m}{ld}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The variations expressed by eqs. (2) and (3) may in the usual manner be represented by straight lines. But when $x=0$ the values of S_A and S_B each become equal to $\frac{-ml-m^2}{ld}$; this means that the straight lines intersect or cross at the origin of ordinates (also origin of moments) where $x=0$. Following this reasoning the influence line for stress in CG may at once be drawn; through any point, as U, in the vertical line through E draw the lines UV and UT to the ends of the span. When $x=m$, $S_A=0$; and when $x=l+m$, $S_B=0$. Hence eqs. (2) and (3) represent the lines UYT and UVW. Dropping the verticals CW and DY and connecting W with Y, the influence line $VWXYT$ at once results, for within the panel CD this line is straight.

Whenever the centre of moments falls within convenient limits on the drawing, Method II is preferable to Method I, as it is general and holds whether the centre of moments falls within or without the limits of the span.

The scale with which to measure the influence line is easily obtained, for, after having obtained the stress in CG for a load placed at any point, such as D, the scale for the remaining ordinates is at once determined.

It should be noted that this analysis is also directly applicable for drawing the influence lines for the stresses in the chord members.

Art. 17.—Criterion to Determine Position of Loading for Maximum Web Stress.

The web member CA of the truss shown in Fig. 12 will be chosen, and it will be supposed that the loads advance upon the bridge, along the lower chord, from right to left. It is at once seen, by inspection of the influence line, that in general no load must pass the panel in question, since loads in that portion of the truss to the left of the panel will always cause negative stress; the exact distance which the loading must advance into the panel is then the quantity which must be determined. Let l be the distance from the end of the span R' to the point D at which a load causes no stress in CA; l' the distance from the right end of the panel to the same point D; x the distance of any load P from the end of the span; x' the distance of any load P from the right end B of the panel; m the general value of the influence ordinate between R' and B corresponding to a unit load; $(m-m')$ the value of the similar quantity between B and C; a the value of m' at the point D; ΣP all the weights on the bridge; $\Sigma P'$ the weights on the panel BC; and S the stress in the member AC. Evidently

$$S=\Sigma Pm-\Sigma P'm'. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From similar triangles there is found that $m=\frac{ax}{l}$ and $m'=\frac{ax'}{l'}$. Substituting these values, eq. (1) becomes

$$S=a\left\{\frac{1}{l}\Sigma Px-\frac{1}{l'}\Sigma Px'\right\}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the train advances to the left by an amount equal to Δx, the stress becomes

$$S'=a\left\{\frac{1}{l}\Sigma P(x+\Delta x)-\frac{1}{l_1}\Sigma P(x'+\Delta x)\right\},$$

and the change in the stress is therefore

$$\Delta S = S' - S = a\left\{\frac{1}{l}\Sigma P \Delta x - \frac{1}{l'}\Sigma P' \Delta x\right\}, \quad . \quad . \quad (3)$$

assuming that no new loads advance upon the truss, and that no new loads enter the panel.

For a maximum or minimum $\Delta S = 0$; hence for this condition

$$\frac{l'}{l} = \frac{\Sigma P'}{\Sigma P}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This equation is a perfectly general equation of condition for finding a maximum stress, and is made applicable for finding the greatest counter stress by substituting for l and l' the proper quantities measured from the left ends of the truss and panel.

In order that eq. (4) may hold, it will usually be found necessary to place a weight at the panel point B, and to consider only so much of it in the panel CB as may be necessary to fulfil the conditions of the equation.

This criterion may be used graphically precisely as in the case of moments in Art. 7. Lay off the weights P_1, P_2, etc., on a vertical line passing through N, Fig. 12, beginning at the bottom with P_1, and on a strip of paper lay off the distances between the loads, beginning at the right with P_1. Let NE represent the total load ΣP on the bridge, P_2 being at the panel point B. Connect D' with E by a straight line cutting a vertical erected at B or P_2 at F. If a horizontal line drawn through F intersects the load P_2 on the line NE the equation of condition is satisfied, and this will be one position of the loading, causing maximum stress. For, from similar triangles,

$$\frac{D'G}{DN} = \frac{NH}{NE} \quad \text{or} \quad \frac{l'}{l} = \frac{\Sigma P'}{\Sigma P}.$$

Application of the Criterion.

An application of this method to find the maximum and minimum stresses in the diagonal S_4 of the truss shown in Fig. 14 will now be made. The following are the required data:

Span = 264 feet; number of panels = 8; panel length = 33 feet; depth of truss at centre = 42 feet; length of $S_3 = 39$ feet; length of $S_1 = 31$ feet; loading = E 40 of Cooper's specifications.

Following the methods of Art. 15, it will be found that D is the point where a load, if placed, will cause no stress in S_4, and this point is projected vertically downward as D' upon the base line MN. It will be supposed that the loading has advanced upon the span from the right, and to such a point that wheel load 3 is at the panel point B. The number of concentrations on the span, as well as the position of each load, is easily found by laying off to scale on a strip of paper the distances between the loads and moving this strip until wheel 3 is at the point G. The line MN shows this particular position of the loads. It should be noted that 10 feet of uniform load are treated as one concentrated load, the effect of the latter being to act at the centre of the 10-foot section. The amounts of the wheel loads are then laid off upward in regular order on the vertical erected at the right end of the base line, extending from N to E. The points D' and E are then connected by a straight line and a horizontal FH drawn through F, the intersection of $D'E$ with a vertical erected at G. Then, since FH intersects the load line in the load 3, the conditions of eq. (4) are fulfilled, and this position of the loading causes a maximum stress.

In order to determine the actual stress in S_4, it will then be necessary to find both the reaction at the left end of the span and the reaction at the left end of the panel; these may be found, both for this position and other positions

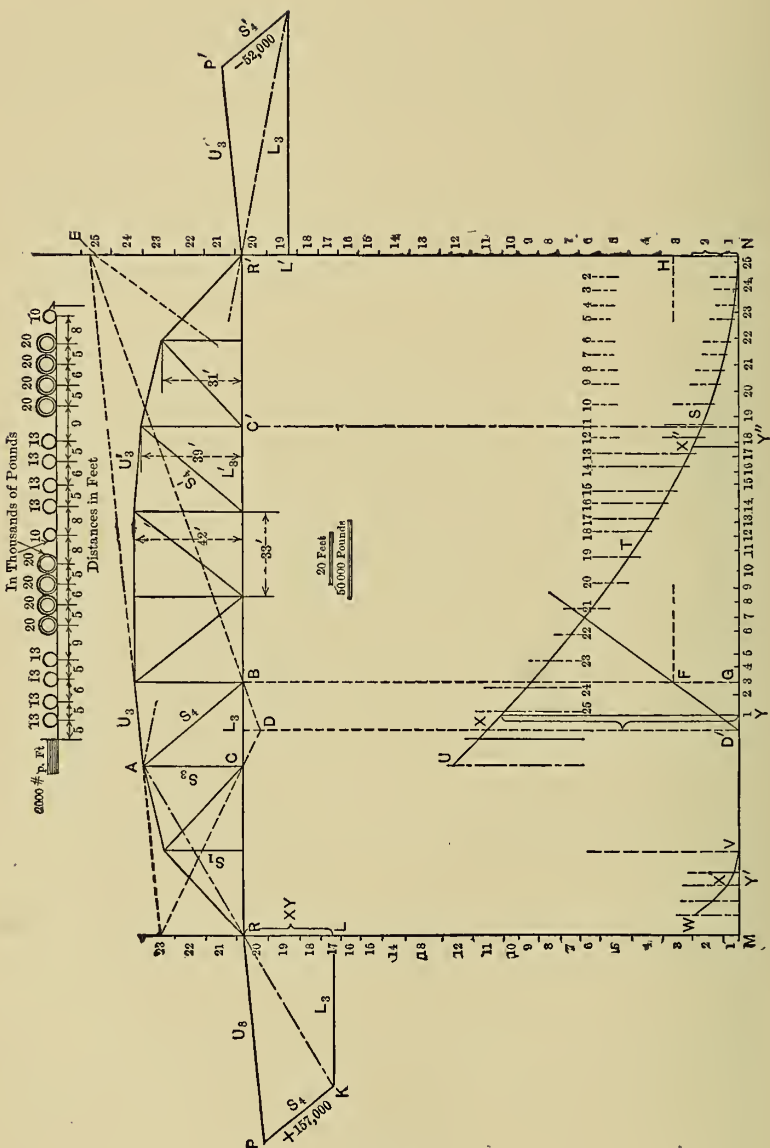

In Thousands of Pounds
Distances in Feet
2000 # p. Ft.
20 Feet
50000 Pounds
+157,000
-52,000

FIG. 14.

which the loading may occupy, from a reaction influence line $NSTU$ for the main span and a line VW, shown in the left-hand corner, for the panel length. These forces XY and $X'Y'$ are the only external forces acting at the left of the section passing through the members U_3S_4 and L_3; therefore their resultant holds in equilibrium the stresses in these three members and these stresses may be found by means of the method of Art. 14.

At this point, however, an approximation involving but small error and that on the side of safety will be introduced and consists simply in neglecting the panel reaction $X'Y'$; that is, instead of taking the resultant of these two external forces, which acts a little to the left of the point R, only the reaction XY at R is taken. The point A, which is the intersection of the lines of application U_3 and S_4 is then connected with the point R, which is the intersection of the lines of application of L_3 and XY, and the force diagram $PRLK$ drawn, the line RL representing to a much reduced scale the ordinate XY found from the reaction influence line below the wheel load 1. By means of the proper scale, the maximum stress in S_4 is thus found to be a tension of 157,000 pounds.

In order to find the minimum stress, the maximum stress in the member S_4' situated in a corresponding position on the other side of the centre of the truss will be found. By means of an exactly similar construction to that employed above, it is found that wheel load 2 at panel point C' will cause a maximum stress. The reaction is found to be $X''Y''$ and the stress by means of the polygon $R'P'K'L'$ at the right end of the stress is found to be a compression of 52,000 pounds. The length $R'L'$ is equal to $X''Y''$. In order to avoid confusion between main and counter stresses, the diagrams for main stresses should always be drawn at the left end, and those for the counter stresses at the right end of the truss.

As already noted, the values of the stresses found in this manner are not quite exact; in order to make them absolutely so, the effect of the small forces acting at the left end of any panel in question must be included and may be treated as if existing independently of the main reaction; the quantity thus found separately must be added algebraically to the stress previously found to make the latter exact. For instance, in the case of S_4 the quantity $X'Y'$ is equal to 6500 pounds acting downward at C; passing a section through U_3, S_4, and L_3 it is found that, due to $X'Y'$, S_4 sustains a compression of 7000 pounds; the exact stress in S_4 is therefore $157{,}000 - 7000 = 150{,}000$ pounds tension. The percentage of error involved in the approximation is never very large; the judgment of the designer must determine whether to apply the exact procedure or whether to make the approximation. Stresses in the other web members may then be found by similar methods of procedure.

There still remain to be determined the stresses in the chords, which are found precisely as in the case of trusses with parallel chords by finding the maximum bending moments at the various panel points and dividing those moments by the proper lever-arms for the various chord members. For instance, to find the maximum stress in L_3 the maximum bending moment for the point A is divided by the distance AC, and the quotient is the stress in L_3. In this way all the live-load stresses may be found; the dead-load stresses, as usual, are to be found by means of the ordinary force polygon.

The preceding work, involving the use of wheel-load concentrations, requires that for every truss a separate reaction influence line $NSTXU$ must be drawn. This requires careful draughtsmanship, and could be avoided if equivalent uniform loads could be fixed upon, for then

the methods of Art. 16 would be simply applied, and influence areas would replace influence lines.

Art. 18.—Trusses with Subdivided Panels.

Trusses of long spans frequently have panels subdivided in the manner shown in Fig. 15. In the case there shown

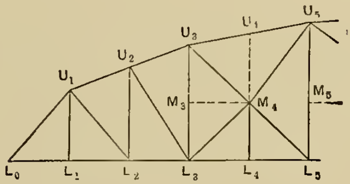


FIG. 15.

the panel L_3L_5 is divided by the tie-rod M_4L_4, which is hung from the point M_4 and which distributes the weight it carries at its lower end to the main panel points of the truss by means of one of several subsidiary trussed panels. In this way, long stringers and consequently heavy floor systems are avoided. The stresses in the truss remain perfectly determinate, but the criteria previously deduced either need some additional explanation or require some modification to be applicable to this form of truss. It should be noted that U_3L_5 is the main web member of the panel L_3L_5 and that U_5L_3 is the counter member, but that portions of each of these web members are also included in the subordinate framing. The members M_3M_4, U_4M_4, etc., shown in dotted lines are not true parts of the truss; their functions are simply to support compression members at intermediate points, and thus by reducing their effective lengths permit the use of higher intensities of stress.

In the treatment which follows it will not be necessary to discuss both the main and counter web members, since

the same methods are applicable to both; it is only necessary to remember that main members have their maximum stresses when the loading covers the longer portion of the truss and counter members when the loading covers the shorter portion of a truss.

Considering, then, only the panel L_3L_5, Figs. 16 to 19, the dotted lines illustrate the various ways in which the

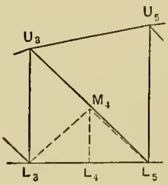


FIG. 16.

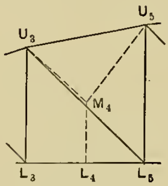


FIG. 17.

subordinate bracing may act to transfer the load at the point M_4 to the panel points L_3, L_5, U_3, and U_5. In Fig. 16 the trussing $L_3M_4L_5$ distributes the load to L_3 and L_5; in Fig. 17 the trussing $U_3M_4U_5$ distributes the load to U_3 and U_5; in the former case the subordinate inclined bracing (shown in dotted lines) is in compression; in the latter, in tension. In Fig. 18, the trussing $U_3M_4L_3$ distributes the load to U_3 and L_3 and similarly in Fig. 19, $U_5M_4L_5$ distributes the load to U_5 and L_5; in these two cases the subordinate bracing is partly in tension and partly in compression. The tie-rod sustains tension in all cases.

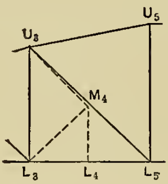


FIG. 18.

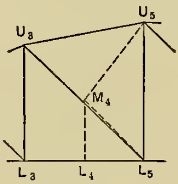


FIG. 19.

Art. 19.—Maximum Web Stresses in Trusses with Subdivided Panels.

It is at once seen that the bracing shown in dotted lines in Figs. 16 and 17 transforms the short stringers L_3L_4 and L_4L_5 into one trussed stringer, and that the loads on the panel length L_3L_5 are distributed only to the points L_3 and L_5. The criterion of Art. 17 may then be immediately applied as it stands, taking L_3L_5 as the panel length. The maximum stress in U_3L_5 will then be found as the tension resulting from the position of the loading determined by this criterion; but in Fig. 16 this stress must be decreased for M_4L_5 by the amount of compression which this portion of U_3L_5 carries as a member of the trussed stringer. Similarly in Fig. 17 the stress in U_3M_4 must be increased by the amount of tension which this member carries as its share of the trussed stringer.

In treating the web member of Fig. 18, it will be necessary to consider the form of the influence line for stress in M_4L_5. Passing a section through U_3U_5, M_4L_5, and L_4L_5, it is at once seen that for a load between the right abutment and L_5, the stress in M_4L_5 varies directly as the left-hand reaction. Similarly for a load between the left abutment and L_4 the stress varies directly as the right-hand reaction. Consequently the criterion of Art. 17 is directly applicable to M_4L_5, the panel length in this case being the short panel L_4L_5. This reasoning applies in exactly the same way to the short panel length L_3L_4 in the case of U_3M_4 of Fig. 19.

The maximum stress in the remaining portion of the member U_3L_5 in either of these two cases is then assumed to be that already determined for the other portion, increased or diminished, as the case may be, by the amount of stress which the part shown in dotted lines must carry

in performing its duty as a part of the subordinate bracing. It is evident, however, that this result does not give the absolute maximum stress that may occur in U_3M_4 (Fig. 18) or M_4L_5 (Fig. 19). The stress in these members is composed of two parts—that part due to the true stress in the web member itself, and that part due simply to the weight carried by the tie-rod. A criterion indicating the position of the loading giving a simultaneous maximum condition of these two factors is not a simple one. As but a small error is involved in assuming that the position of the load causing maximum stress in one portion of the web member will also cause the maximum stress in another part, the same position of the loading is generally assumed for both portions of the web member.

These investigations indicate that the maximum web stresses in trusses with subdivided panels may be obtained by means of methods previously deduced.*

Art. 20.—Maximum Chord Stresses in Trusses with Subdivided Panels.

Unloaded Chord.

The stress in any unloaded chord member such as U_3U_4 (Fig. 20) is found by passing a section through U_3U_4, U_3M_4, and L_3L_4, and with the centre of moments at L_5, equating the moment of the external forces situated on one side of the section with the moment due to the chord stress. Although the centre of moments is at L_5, the only external panel forces on one side of the section are L_1 to L_3, the load at panel point L_4 not being included.

* The two cases of Figs. 18 and 19 are recognized in order to complete the treatment, although it is doubtful whether an entire panel load can act as shown by the dotted lines of those figures.

This invalidates therefore the use of the simple criterion,

$$\frac{\Sigma P}{\Sigma P'} = \frac{l}{l'},$$

deduced in Art. 6. By methods parallel to those of that article, however, the proper criterion may be quickly deduced.

The influence line for the stress in U_3U_4 is easily drawn, for as a load passes from the left end of the truss to panel

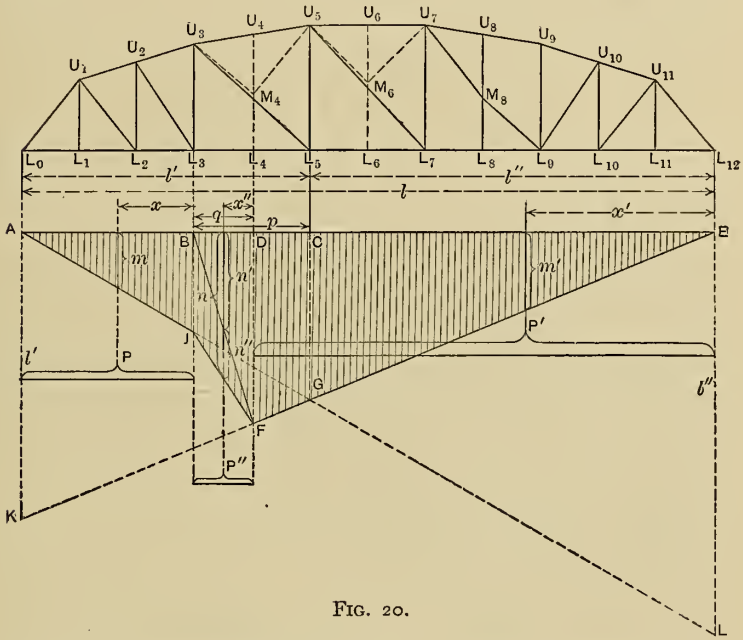


FIG. 20.

point L_3, the stress varies directly as the right-hand truss reaction, i.e., as a straight line. A proper slope for this line may be indicated (as in the case of moments, Art. 5) by laying off EL equal to l'' on the left end of the reference line and connecting L with A, the right end of the reference line.

In a similar manner, the line EF indicates that between L_0 and L_4 the stress also varies as a straight line, and its slope is found by laying off AK equal to l' at the left end of the reference line.

Since all influence lines for single loads are straight lines between panel points, the influence line may then be closed by the line JF.

The notation which will be employed to develop the criterion is indicated in the figure. P, P'', and P' represent in general any load between L_0 and L_3, L_3 and L_4, and L_4 and L_{12} respectively, while m, n, and m' represent the corresponding influence ordinates.

In general, then, the stress in U_3U_4 may be represented by the following expression:

$$S=\Sigma(P\cdot m)+\Sigma(P''\cdot n)+\Sigma(P'\cdot m'). \quad . \quad . \quad (1)$$

The ordinate n may be divided by the line BF; then

$$\Sigma(P''\cdot n)=\Sigma(P''\cdot n')+\Sigma(P''\cdot n''). \quad . \quad . \quad (2)$$

By substituting this value of $\Sigma(P''\cdot n)$ in eq. (1) and by replacing the values of the influence ordinates by equivalent values found from similar triangles, the value of S becomes

$$S=\Sigma\left[P\cdot\frac{BJ(AB-x)}{AB}\right]+\Sigma\left[P''\cdot\frac{DF(q-x'')}{q}\right]$$
$$+\Sigma\left[P''\cdot\frac{BJ\cdot x''}{q}\right]+\Sigma\left[P'\cdot\frac{CG\cdot x'}{CE}\right]. \quad . \quad (3)$$

By moving the loads an infinitesimal amount to the left the change in the stress or ΔS becomes

$$\Delta S=\Sigma\left[P\cdot\frac{BJ\cdot\Delta x}{AB}\right]+\Sigma\left[P''\cdot\frac{DF\cdot\Delta x}{q}\right]$$
$$-\Sigma\left[P''\cdot\frac{BJ\cdot\Delta x}{q}\right]-\Sigma\left[P'\cdot\frac{CG\cdot\Delta x}{CE}\right]$$

$$=\Sigma\left[P\cdot\frac{l''}{l}\cdot\Delta x\right]+\Sigma\left[P''\cdot\frac{l''+p-q}{lq}\cdot\Delta x\right]$$

$$-\Sigma\left[P''\cdot\frac{l''(l'-p)}{l\cdot q}\Delta x\right]-\Sigma\left[P'\cdot\frac{l'}{l}\cdot\Delta x\right]. \quad . \quad (4)$$

The condition for maximum stress requires that ΔS be placed equal to zero. By simplifying terms and by noting that

$$\Sigma\left(P\cdot\frac{l''}{l}\right)=\Sigma\left(P\cdot\frac{l}{l}\right)-\Sigma\left(P\cdot\frac{l'}{l}\right),$$

there will be found that

$$\Sigma\left[(P+P'+P'')\cdot\frac{l'}{l}\right]=\Sigma(P)+\Sigma\left(P''\cdot\frac{p}{q}\right). \quad . \quad (5)$$

Eq. (5) is similar in form to eq. (5) of Art. 11, the latter being the criterion for finding the maximum bending moment at any point in any truss. It differs, however, in this essential, that in the present case the factor to be applied to the loads within the panel is p/q and not q/p.

The position of the loading having been determined by means of this criterion, the stress in U_3U_4 is found by taking moments about L_5. The maximum stress in U_4U_5 occurs with the same position of loading as U_3U_4, and its value is also the same.

Loaded Chord.

The stress in any loaded chord member such as L_3L_4 is found by passing a section through U_3U_4, U_3M_4, and L_3L_4, and taking the centre of moments at U_3. This permits the use of the simple moment criterion,

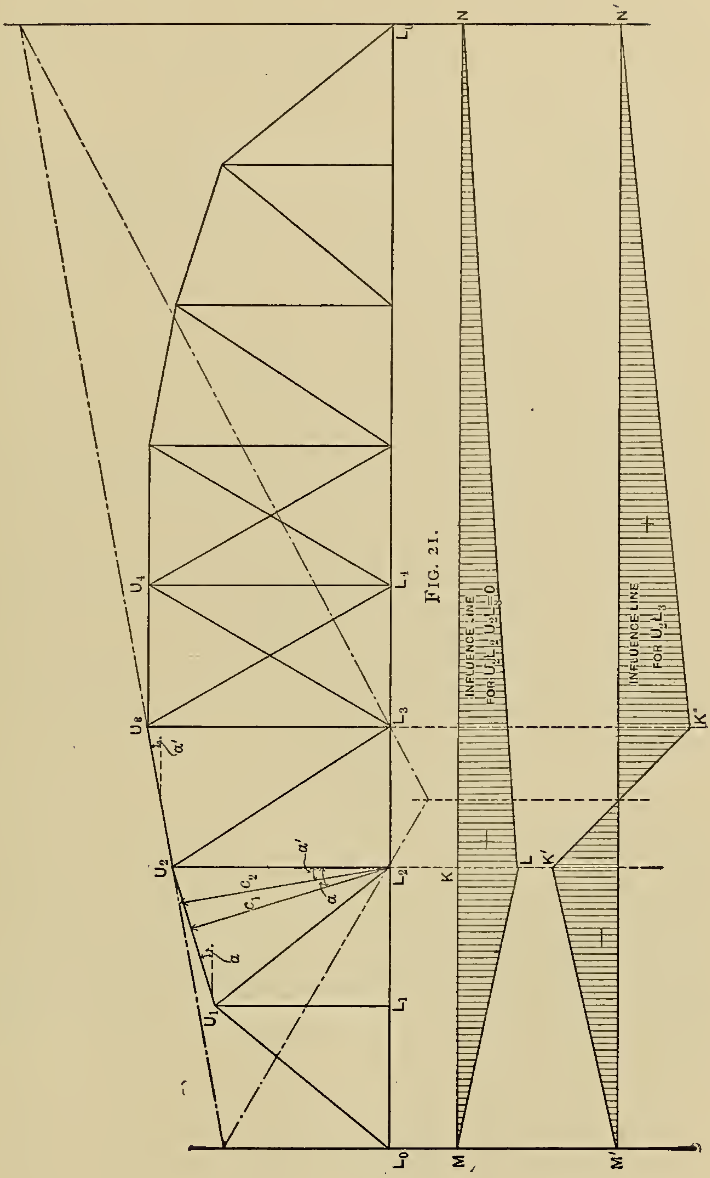
$$\frac{\Sigma P}{\Sigma P'}=\frac{l}{l'}$$

of Art. 6, and no unusual conditions are encountered in finding the stress in L_3L_4. The maximum stress in L_4L_5 is found with the same position of loading as L_3L_4, and the value of its stress is exactly the same.

Art. 21.—Counter-Stresses in a Vertical Post at an Angle in a Chord.

Although the usual stress in the vertical post of a truss with inclined upper chords is compressive, it is possible for such a member to receive a counter tensile stress. This counter stress is not caused by the negative shear in a panel, but it occurs when the inclined web member, cutting the upper end of the vertical at the unloaded chord, sustains a small or even zero stress. In that case the equations of equilibrium, as applied to the unknown forces at the panel point in question, involve only the stresses in the two chord members and in the vertical post. Since the axes of the two chord members meeting at that point are not in a straight line, a component in the direction of the post must result. In a through truss with horizontal lower chord this component furnishes tensile stress. It becomes necessary, therefore, in order to determine this maximum tension in these posts to find that position of the loading which causes a zero stress in the inclined member; that is, the live load must cause a stress in this inclined member equal and opposite to that of the dead load. This position is easily determined by the use of the influence line.

Assuming for the present that the position of the loading causing the maximum tension in the post is known, it becomes necessary to deduce an expression for the tensile stress in that member. Let Fig. 21 represent a truss in which it is desired to find the tension in the member U_2L_2 when the stress in U_2L_3 is zero. It will be convenient



L0
L1
L2
L3
L4
U1
U2
U3
U4
M
N
K
L
M′
N′
K′
K″
INFLUENCE LINE FOR U1L2 U2L3=0
INFLUENCE LINE FOR U2L3

FIG. 21.

FIG. 22.

first to determine the stress in U_2L_2 for a load P placed at panel point L_2. The following notation will be used:

p = panel length;
h = the height at the panel under consideration—in this case the length U_2L_2;
d_1 = the difference in the height of U_1L_1 and U_2L_2;
d_2 = the difference in the height of U_2L_2 and U_3L_3;
$d = d_1 - d_2$;
l = the length of truss;
l_1 = the length of the member U_1U_2;
l_2 = the length of the member U_2U_3;
P = the load at the panel point L_2;
n = the number of panels (of equal length) between the left end of the truss and the member U_2L_2;
c_1 = the lever-arm of U_1U_2 about L_2;
c_2 = the lever-arm of U_2U_3 about L_2;
α and α' = the angles of inclination of U_1U_2 and U_2U_3 with a horizontal respectively;
R = the reaction at left end of span.

Then for the load P at L_2,

$$R = \frac{(l - np)P}{l}.$$

The stress in U_2L_2 may be determined by finding, first, the stresses in U_1U_2 and U_2U_3 and taking the difference of their vertical components as the stress in U_2L_2. By taking moments about L_2 and omitting the stress in U_2L_3 (which is assumed zero), the stresses in U_1U_2 and U_2U_3 will be

$$U_1U_2 = \frac{R \cdot np}{c_1};$$

$$U_2U_3 = \frac{R \cdot np}{c_2}.$$

Their vertical components are then respectively

$$\frac{R\cdot np}{c_1}\cdot\sin\alpha \quad \text{and} \quad \frac{R\cdot np}{c_2}\cdot\sin\alpha'.$$

The tensile stress in U_2L_2 is the difference of these vertical components:

$$U_2L_2=R\cdot np\left(\frac{\sin\alpha}{c_1}-\frac{\sin\alpha'}{c_2}\right). \quad . \quad . \quad . \quad (1)$$

But

$$\sin\alpha=\frac{d_1}{l_1} \quad \text{and} \quad \sin\alpha'=\frac{d_2}{l_2};$$

$$\therefore\ U_2L_2=R\cdot np\left(\frac{d_1}{l_1\cdot c_1}-\frac{d_2}{l_2\cdot c_2}\right). \quad . \quad . \quad . \quad . \quad (2)$$

Also

$$\frac{c_1}{p}=\frac{h}{l_1} \quad \text{and} \quad \frac{c_2}{p}=\frac{h}{l_2}.$$

Therefore eq. (2) takes the form

$$U_2L_2=R\cdot np\left(\frac{d_1}{h\cdot p}-\frac{d_2}{h\cdot p}\right)$$

$$=\frac{R\cdot np\cdot d}{h\cdot p}=\frac{R\cdot n\cdot d}{h}. \quad . \quad . \quad . \quad . \quad (3)$$

Eq. (3) furnishes the expression by whose aid the influence line for the tensile stress in the number U_2L_2 may be drawn. As the stress in U_2L_2 varies directly as R the variation in stress for loads at other panel points may be represented by the ordinates between the base line MN and the straight lines drawn from the end of the ordinate KL below L_2 to M and N, as in Fig. 22. It should be noted that this influence line, although for a

web member, shows no reversal of stress throughout its length; it is to be used *only* when the stress in U_2L_3 is zero. The scale of Fig. 22 is found by the aid of eq. (3), which furnishes the value of the ordinate KL, the stress in U_2L_2, when the load P is placed at L_2.

In order to determine the position of the load which causes the maximum tension it is necessary to use the influence line for stress in the member U_2L_3. This line is represented by $M'K'K''N'$ in Fig. 22, the method of deriving it being that of Art. 16. The loading must advance from the left toward the right until it is found by trial that the live-load stress in U_2L_3 is just equal to the dead-load stress found previously. The stress in U_2L_2 for this position of the loading may then be determined from the influence line drawn for that member. It is usual in the treatment of this counter-stress to substitute a uniform live load for the actual locomotive concentrations. The results thus obtained are not rigorously exact, but to use locomotive-wheel loads would involve needless refinement.

One other point, however, remains to be considered, namely, that it may be possible to obtain a zero stress in the member U_2L_3 with a position of the loading that would cause greater tension in the member U_2L_2 than for the position just determined. This position of the loading is that caused by trains entering simultaneously upon the bridge from the two ends, but with a greater load on the left end than on the right end. In that case the stresses in U_2L_3 caused by the two different loadings tend to neutralize each other, but the left-hand loading must be so much greater as to balance the dead-load stress. Such a position causes greater tension in the member U_2L_2, since according to the influence line for U_2L_2 every additional load on the truss increases its stress.

Example.

As an example, the maximum tension in the member U_3L_3 of the truss detailed in Chapter VI and shown in Fig. 23 will be determined, the locomotive concentrations being replaced by a uniform live load of 2500 pounds per linear foot of truss.

For that case the dead-load stress in U_3M_4 has been found to be $+90,000$ pounds; therefore the live load must

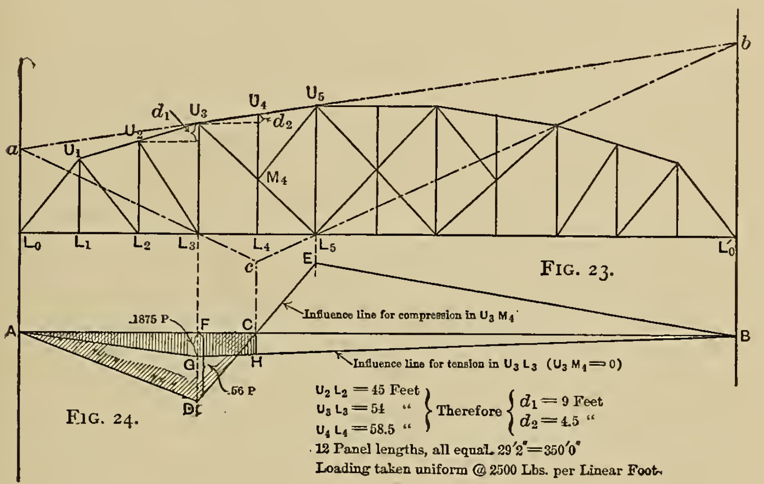


FIG. 23.

FIG. 24.

advance on the truss from the left end, until the stress caused by it in U_3M_4 is just equal to $-90,000$ pounds. The influence line for U_3M_4 is therefore drawn in the usual manner (Fig. 24), as follows: The chord member U_3U_4 is continued until it intersects verticals through the abutments at a and b. Lines are then drawn from these points through the ends of the panel L_3 and L_5 to intersect at c. This point is projected to the base line AB to C, where it indicates the position of a load on the truss causing no stress in U_3M_4 (Art. 16). Placing a load of

unity at L_3, it may then be found that the stress in U_3M_4 is -0.56; the influence ordinate FD is therefore drawn with that value. The completed line for U_3M_4 is then $ADCEB$.

It will now be found by trial that the area ADC, which represents compressive stress, has the following value, if the length AC is entirely covered by the uniform load of 2500 pounds per linear foot:

$$2500\left(\frac{AF\times FD}{2}+\frac{FC\times FD}{2}\right)=2500\left(\frac{AC\times FD}{2}\right)$$

$$=2500\left(\frac{116\times .58}{2}\right)=-81,200 \text{ pounds.}$$

It is seen that this stress does not quite equal the tension due to the dead load, but since its value is so nearly the same, and since uniform instead of concentrated loading has been used, it will be proper to assume that this position of the loading furnishes zero stress in U_3M_4.

The influence line for U_3L_3 is then drawn. Placing a unit load at L_3, eq. (3) will furnish the following value, remembering that $d=4.5$ feet, $n=3$, $h=54$ feet, and $R=0.75$:

$$U_3L_3=\frac{.75\times 3\times 4.5}{54}=+.1875.$$

The influence line for U_3L_3 is then AGB, the ordinate FG having a value of .1875. The tension in U_3L_3, for the loading just found, is therefore

$$U_3L_3=2500 \text{ (area } AGF+\text{area } FGCH).$$

Since

$$GF=.1875 \qquad FC=28.5 \text{ feet}$$
$$AF=87.5 \text{ feet} \qquad CH=.165$$

$$\therefore\ U_3L_3=+26,750 \text{ pounds.}$$

Art. 22.—Influence Line for Stress in the Web Member of a Truss when the Centre of Moments Falls within the Limits of the Truss.

The influence lines for stresses in web members deduced in the previous articles are applicable only to those trusses in which the centre of moments falls outside of the limits of the truss. In the case of the web member U_1L_2, shown in Fig. 25, the influence line may be determined in the

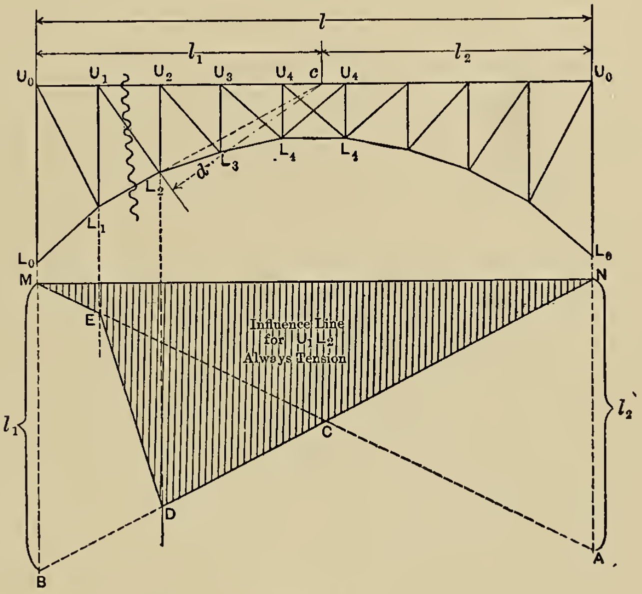


FIG. 25.

following manner. The stress in this member, U_1L_2, is found by passing a section through the members U_1U_2, U_1L_2, and L_1L_2 and dividing the sum of the moments of all the external forces (not shown) on one side of this section about c, the centre of moments by the lever-arm d of U_1L_2 about c.

Assuming a load of unity to pass between the left end of the truss U_0 and the left end of the panel U_1 cut by the section, the stress in the member may be expressed as

$$\frac{x \cdot l_2}{l \cdot d},$$

in which x represents the distance from the left abutment. Since the only variable in this expression is x the variation of the stress may be represented by a straight line, x being of the first degree. The stress is tensile. Similarly, for a load of unity between the right end abutment U_0 and U_2, the right end of the panel cut by the section, the variation in the stress may be represented by the straight line

$$\frac{x \cdot l_1}{l \cdot d},$$

assuming x now to be the distance from the right abutment. The stress is again tensile.

These two expressions are proportional to l_2 and l_1 respectively. Erecting MB at the left end of the reference line MN equal to l_1 and NA at the right end equal to l_2 and connecting B and A to N and M respectively, the lines ME and DN will form part of the desired influence line. Since it has been shown (Art. 10) that influence lines between panel points are straight lines, it then only becomes necessary to connect the points E and D by a straight line to complete the diagram.

It is therefore seen that in the case of a web member of the kind shown in the figure, counter-stress never occurs. The member sustains its maximum stress with the entire truss covered with load. A similar construction might be developed for the vertical members. It is possible to deduce a criterion for maximum stress in the member

U_1L_2 by means of the influence line shown in the diagram, but since this form of truss is rarely built such a criterion need not be developed. The treatment of an equivalent uniform load is, moreover, so simple that it should be used to replace the locomotive concentrations. For that character of loading it is only necessary to multiply the area of the figure $MEDCN$ by the proper intensity of the uniform load to obtain the maximum stress.

Art. 23.—Influence Lines for Skew Bridges.

Skew bridges are trussed structures in which the ends of the pair of trusses forming the bridge do not lie in a line perpendicular to its axis. The skew of a bridge is the distance between the projections of the ends of the trusses measured on the axis of the bridge. This distance is not necessarily the same at the two ends. In Fig. 28, which represents the plan of the lower chord system of a skew bridge, the skew at the left end of the bridge is a full panel length, or $L_0'L_1'$, while at the right end of the bridge it is $eL_7 = cd$. This example illustrates a skew bridge in the most general form.

It is customary to place the floor-beams, supporting the floor system, at right angles to the trusses, as shown in the figure, and for convenience in the manufacture of the details of the portal bracing it is preferable to design the end posts of opposite trusses with the same inclination. This end is attained by moving the upper end of the end post of the further truss, that is, U_6' in Fig. 27, half the skew distance back of the vertical projection of L_6', while U_6, Fig. 29, is moved half the skew distance in advance of the vertical projection of L_6. The members $U_6'L_7'$, Fig. 27, and U_6L_7, Fig. 29, will then be parallel. The

upper chord members $U_5'U_6'$ and U_5U_6 will not be equal in length, nor will $U_6'L_6'$ and U_6L_6 be either parallel or

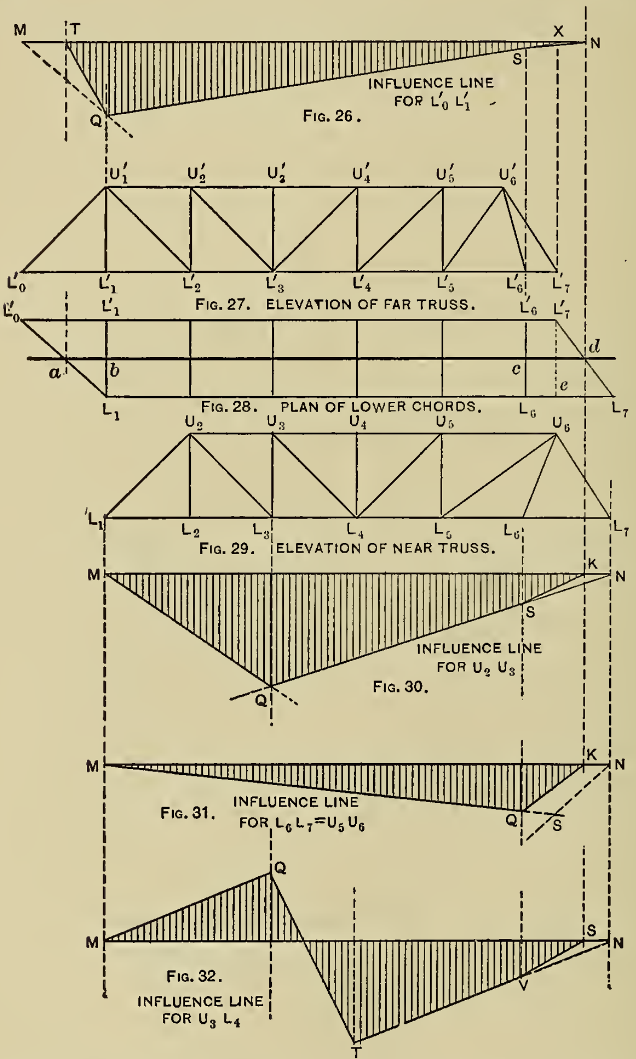


FIG. 26.

FIG. 27.

FIG. 28.

FIG. 29.

FIG. 30.

FIG. 31.

FIG. 32.

vertical; but this is preferable to having the end posts non-parallel.

At the left end of the bridge, such construction in the example considered is not necessary, for the skew distance is a full panel length.

Should the skew distances at the two ends of the bridge be equal, the two trusses would be exactly alike except that they would be turned end for end.

The moving load is taken to pass along the centre line *ad* of the bridge, and this necessitates changes in the influence lines for stresses in the various members. Treating first the member U_2U_3 of the truss of Fig. 29, it is seen that its stress will vary as the moment at L_3. If the truss $L_1 \ldots L_7$ carried loads on the span $L_1 \ldots L_7$, the moment influence line at L_3 would be MQN, Fig. 30. By referring to Fig. 28, however, it will be seen that loads between L_1 and L_6 only are carried on that span length. Loads are carried on the line *cd* only in the panel L_6L_7, and a load at *d* causes no stress in the truss L_1L_7. Since the influence line is straight between all panel points, the line SK, Fig. 30, will exhibit the variation of the moment at L_3 for loads between *c* and *d*. The final influence line is therefore $MQSKN$. For uniform loading it is only necessary to measure the area shown shaded, and to multiply by the proper intensity of the loading to obtain the value of the moment. For concentrated loads the criterion previously deduced would require modification for the portion SK of the line, although ordinarily no serious inaccuracy results from the use of the general formula. The values of the moments for different positions of such concentrations may also be found by trial and comparison.

The treatment of chord member L_6L_7 or U_5U_6 is shown in Fig. 31. If the truss had the span length L_1L_7, the influence line would be MSN, the centre of moments being at U_6; but owing to the skew, the final line becomes MQK.

It should be noted that in this truss the stress in L_5L_6 is not equal to the stress in L_6L_7 if there be a load at L_6, for

the member U_6L_6 adds a horizontal component at panel point L_6.

The web members are also treated in precisely the same way. Fig. 32 illustrates the treatment of U_3L_4, for which the final influence area is shown shaded.

The treatment of the further truss, Fig. 27, does not differ from the preceding. The influence line for the member $L_0'L_1'$ is shown in Fig. 26. If the truss had the span length $L_0'L_7'$, the influence line would be MQX, M and X being vertically above the ends L_0' and L_7' of the truss, and Q being above the centre of moments. The loads advance along the centre line ad, however, and those at a and d cause zero stress in the truss. The true influence line for $L_0'L_1'$ is therefore $TQSN$, Fig. 26, and the influence area for uniform loading is shown shaded.

The preceding treatment is entirely general, and may be applied to any skew bridge.

Art. 24.—Influence Lines for Double-intersection Trusses.

The influence line possesses distinct advantage in the treatment of double-intersection trusses whether with parallel or broken chords, although in the example treated, Fig. 33, a truss with parallel chords only will be considered. The usual analytical method of treatment is to assume that the structure is composed of two systems of trussing, Figs. 34 and 35, acting independently. The maximum stresses that can occur in each system are found by trial, and if the same member acts in both systems, the sum of the stresses found is taken, provided both have been found for the same position of loading. Conditions of loading which fail to cause simultaneous stresses in the two systems must not be considered in such summations of stresses.

In the treatment of chord members the entire structure

is covered by load, and its position must not be changed when considering the same member for the two systems.

In the graphical treatment it is only necessary to consider that a load at a panel point is carried entirely by the system of which that panel point is a part. If the load is within a panel, it is distributed in some manner between the two systems.

The truss shown in Fig. 33 contains an even number of panels and avoids a possible ambiguity which might be caused in two systems of trussing not symmetrical about a vertical centre line, i.e., equal loads placed at equal distances from the centre of span would rest on two different systems. The treatment by influence lines avoids this ambiguity.

Chord Members.

Let it be required to find the influence line for stress in the member U_4U_5, Fig. 33. This member forms part of U_4U_6 of Fig. 34 and part of U_3U_5 of Fig. 35. The influence line for U_4U_6 is *MRN* of Fig. 36, the point *R* being the projection of the centre of moments L_6. Similarly, *MSN* is the influence line for U_3U_5, the centre of moments being at L_5. It has been assumed that a load at a panel point acts only in the system to which that panel point belongs. A load at L_4, then, causes a stress which may be represented by the ordinate below it enclosed by the line *MRN*; and this applies to all the lower panel points of Fig. 34. A load at L_1 is, however, taken as divided equally between the two systems of trussing. Similarly, loads at the lower panel points of Fig. 35 causes tresses represented by ordinates enclosed within the line *MSN*. But all influence lines between panel points are straight lines. Hence the final influence line for U_4U_5 is represented by *MabcdSRghijkN*, and for uniform loading the stress would be obtained by multiplying the area shown shaded by the proper intensity of loading. For reasons already explained, the points *a*

and *k* lie midway between the two original influence lines drawn. It is evident that for chord stresses no serious

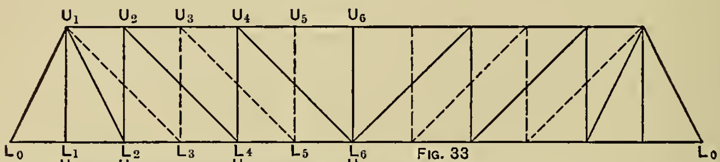


FIG. 33

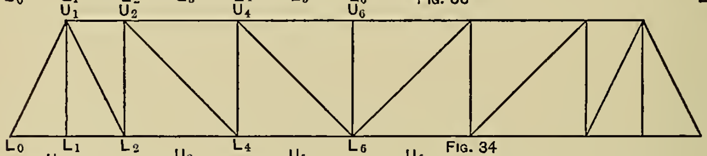


FIG. 34

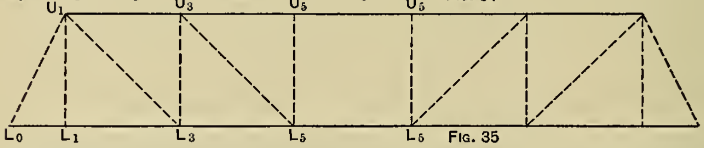


FIG. 35

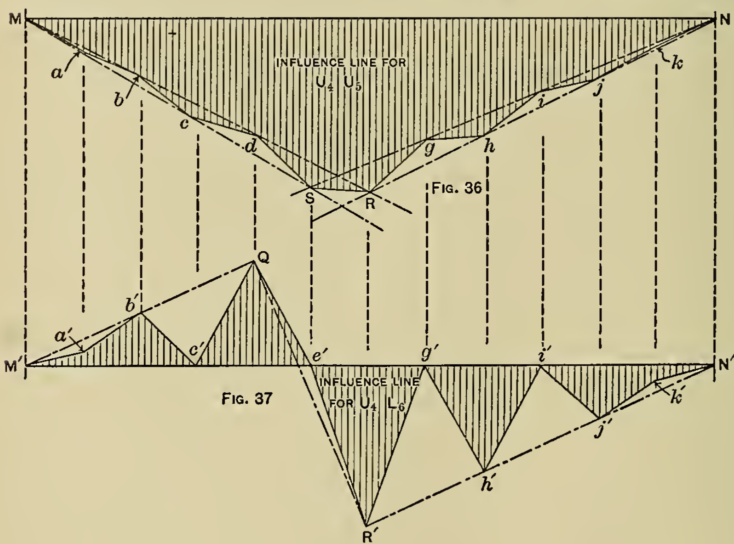


FIG. 36

FIG. 37

error is involved, if the area of either of the first two influence lines be measured. The choice must, however, be left to the designer's judgment.

Web Members.

The treatment of web members follows precisely that of chord members. In Fig. 37 the line $M'QR'N'$ represents the influence line for the member U_4L_6, treating it only as a member of the system of trussing shown in Fig. 34. The points Q and R' are projections of the panel points L_4 and L_6. Since a load at a panel point of the other system of trussing causes no stress in U_4L_6, the projections of those panel points on the base line $M'N'$ all represent zero stresses; that is, the base line $M'N'$ is the influence line for stress in U_4L_6, when the loads are placed at the panel points of Fig. 35. The final influence line is therefore $M'a'b'c'Q'e'R'g'h'i'j'k'N'$. The points k' and a' lie, as before, midway between the two influence lines.

CHAPTER III.

THE THREE-HINGED ARCH.

Art. 1.—To Pass a Funicular Polygon Through Three Points.

LET it be required to pass a funicular polygon through the three points A, B, and C, Fig. 1, for the loads P_1, $P_2 \ldots P_7$ shown. The loads $P_1 \ldots P_3$, situated between the points A and B, will be considered separately from the loads $P_4 \ldots P_7$ situated between B and C. Any other division of the loads might be made, but it will usually be most convenient to have the centre point B divide the loads.

In Fig. 2 the force polygon $P_1P_2P_3$ is drawn, and the closing line furnishes the value of a resultant which will balance the loads. Choosing any point, such as O', as a pole, the rays 1, 2, 3, and 4 are drawn and transferred to Fig. 1 to form the funicular polygon 1, 2, 3, and 4. Through the points A and B of that figure lines R and R_1 parallel to the resultant of P_1, P_2, P_3 are first drawn. The intersections of the bars 1 and 4 with those lines then determine the bar 5. The direction of this bar is then transferred to Fig. 2 and drawn through the pole O', thus determining the values of the reactions R and R_1 acting at the points A and B, respectively, for the loads between A and B.

Similarly, choosing as a pole O'', the reactions R_2 and R_3 for the loads between B and C are found for the points B and C. The conditions of the problem require the

funicular polygon 1, 2 . . . 5 to pass through the points *A* and *B*; that is, the bar 5 should be the line 5′ connecting *A* and *B*. Transferring therefore the direction of 5′ to Fig. 2 and drawing it through the point *m*, it will be evident

FIG. 1.

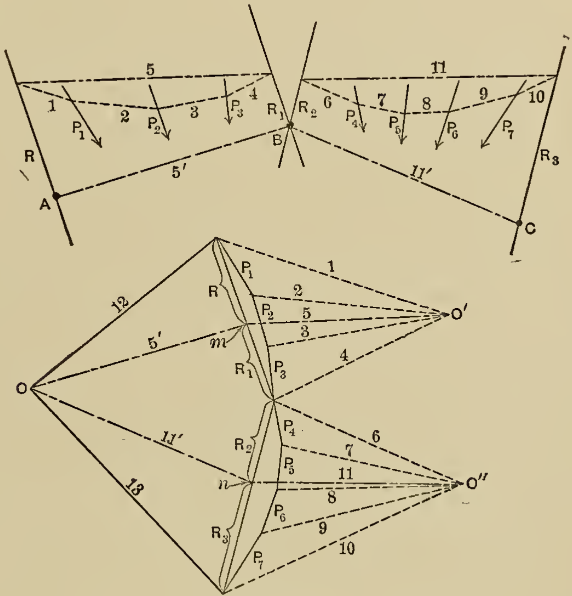


FIG. 2.

that any point on this line will furnish a funicular polygon for the loads $P_1 \ldots P_3$ which will pass through *A* and *B*. In precisely the same way, then, the line 11′ may be drawn in Fig. 2 for the forces $P_4 \ldots P_7$, 11′ being parallel to the line connecting the points *B* and *C*. Any point on this line will furnish a pole for a funicular polygon passing through the points *B* and *C* and constructed for the forces situated between them. Consequently, the intersection of the lines 5′ and 11′ will furnish the only pole which will provide a funicular polygon passing through the three points

A, B, and C for all the forces shown. This polygon is not shown in the figure, but after the pole O is found its construction is obvious.

The rays 12 and 13, being the extreme rays for the true polygon, represent bars in a framework holding in equilibrium the given forces $P_1 \ldots P_7$. They pass through the points A and C respectively, and, as will presently be seen, are the reactions for a three-hinged arch having hinges at A, B, and C and holding in equilibrium the forces given.

Art. 2.—Determination of the Reactions of a Three-hinged Arch.

An arch provided with three hinges is a structure which is statically determinate and in the treatment of the stresses in its members involves only principles already established.

Fig. 3 illustrates a three-hinged spandrel-braced arch provided with hinges at each abutment and at the crown, panel point 13. It is desired to find the stresses in the structure for a fixed position of the loading.

The dead loading, carried at the upper chord panel points, and assumed to have a value of 28,300 pounds per panel per truss, will be treated. The end-panel loads are each taken at 14,150 pounds only. The supporting reactions at the abutments for this type of truss are no longer vertical. Their points of application are given, but their directions are unknown. Since they hold in equilibrium the loading on the truss, the problem becomes that of determining the amounts and directions of two forces whose points of application, 1 and 14, are given, and which hold in equilibrium a known set of forces. The problem would evidently be statically indeterminate except for the condition imposed by the centre hinge, which is that the sum of the moments of the external forces on

either side of the centre hinge must equal zero about that point. The solution of the problem, then, consists in passing a funicular polygon through the three hinges (see Art. 1), the pole distance of that polygon representing the horizontal component of each reaction. This follows,

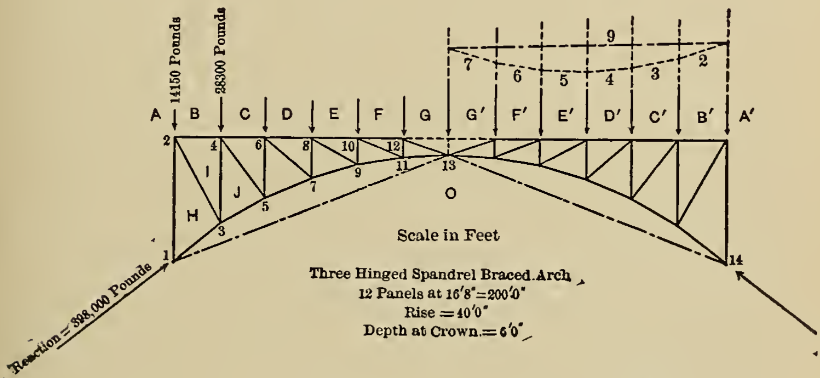


FIG. 3.

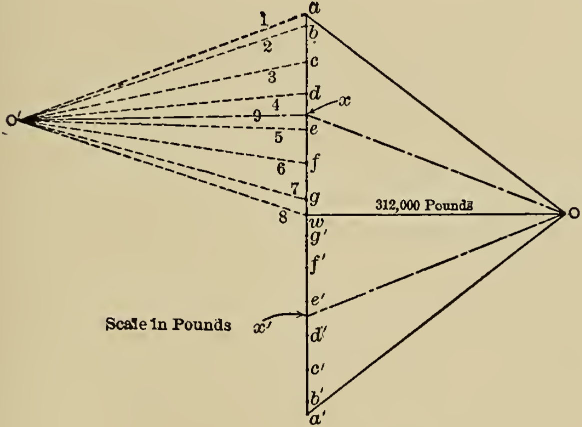


FIG. 4.

since a funicular polygon represents bars in a structure which carry direct stress only for the given system of loads, and if a bar of this funicular polygon passes through any

given point, such as a hinge, there will be no bending moment at that point. Therefore a funicular polygon through the three hinges will furnish the values of the stresses in the supporting or end bars, which are the reactions.

In Fig. 4, therefore, O' is chosen arbitrarily as any pole for the loads on the left half of the structure. The polygon 2, 3, 4 . . . 9 in Fig. 3 is then drawn, and the last ray, viz., 9, transferred to the force diagram, Fig. 4. This determines the point x in the line of the loads, from which point the line xO is drawn parallel to the line connecting the two hinges 13 and 14. Any point on this line xO will furnish a pole from which a funicular polygon passing through the points 13 and 14 may be obtained.

Since the loading is symmetrical about the centre it will not be necessary to complete the graphical operations for the right half of the structure, but the line $x'O$ may at once be drawn parallel to the line connecting the hinges 1 and 13, w being situated midway between x and x'. The intersection of xO and $x'O$ determines at once the true pole of a funicular polygon which will pass through the three points 1, 13, and 14 of the arch. The pole distance is, then, Ow, and it represents a horizontal component in the reactions of 312,000 pounds. The reactions themselves are then compounded from this horizontal force and the vertical reactions aw and $a'w$; they are graphically represented by Oa and Oa', and are each equal to 398,000 pounds.

The horizontal component H may perhaps be more quickly determined by analytical methods, for by taking moments about the centre hinge 13 the following equation will result:

$$H \times 40 = 242{,}950 \times 100 - 14{,}150 \times 100 - 41{,}600[16\tfrac{2}{3}(5+4+3+2+1)];$$

$$\therefore\ H = 312{,}000 \text{ pounds.}$$

This value checks that found graphically.

After the reactions have been determined, the problem presents no further difficulties so far as the determination of the stresses in the members is concerned. The stresses in the members meeting at panel point 1 are first to be found, then those meeting at panel point 2 are next to be treated, and the other panel points in the numerical order shown in the diagram.

The treatment of the stresses has thus far involved a fixed position of the loading, but the positions of the loading causing the greatest stresses in the various members may easily be determined by the aid of the influence lines. In the case of three-hinged roof-trusses the graphical methods thus far outlined are sufficient to find all the desired stresses, for in such cases the loading, in whatever manner it may be applied, is always fixed in position.

Art. 3.—Moments in Three-hinged Arches.

The graphic determination of the reactions of a three-hinged arch, as found in the previous article, is not as simple as the analytic determination, and hence the treatment of a three-hinged arch by the combination of algebraic and graphic methods is more convenient than by the graphic process alone.

The application of influence lines to these structures is of distinct value, since by their aid the positions of loading causing maximum stresses in the various members may be at once determined. For this class of structure, the use of wheel-load concentrations constitutes, however, unnecessary refinement and such loading will not be considered in detail. It is sufficient to state again that if the influence lines for stresses have been drawn for a unit load P, the maximum stress caused by any series of concentrations may be easily determined by trial. The general

value of the stress for a series of loads would then be represented by $\Sigma P \cdot n$, where P represents the value of the load at any point and n the corresponding ordinate.

Before considering influence lines for three-hinged arches it will be necessary to obtain a general expression for the

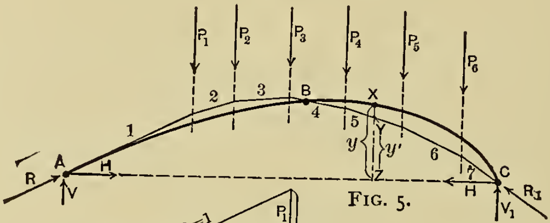


FIG. 5.

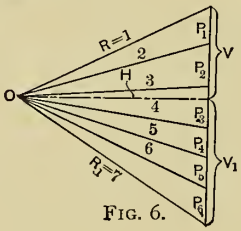


FIG. 6.

bending moment at any section of a three-hinged arch for any system of loading. Let Fig. 5 represent an arch hinged at the points A, B, and C, and let the bending moment at the point X be required.

The reactions R and R_1 may be found by means of the funicular polygon, their values corresponding respectively to the rays 1 and 7 in the force diagram. The horizontal and vertical components of these reactions are represented by H, V, and V_1 respectively. Taking moments of all the left-hand external forces about X, the general expression for M becomes

$$M = \Sigma V \cdot x + H \cdot y, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where V represents the vertical components both of the reaction and of the given loads, x their respective lever-

arms, H the horizontal component of the reaction, and y its lever-arm. But it is evident that the first term in the right-hand member represents the bending moment due to vertical loading at the section X for a simple non-continuous beam of span AC, and its graphical representation is the product of the intercept YZ by the pole distance H or $y' \cdot H$. The second term in the left-hand member of eq. (1) is the product of the same pole distance by a different intercept, viz., $XZ = y$. The bending moment at any section of the arch is, therefore, graphically represented by $H(y - y')$, or the product of the pole distance, H, by the vertical intercept between the point taken on the arch and the true funicular polygon. It is then seen that the line of the arch for the figure shown is subjected at its various points to positive moments for the right-hand section, BC, and negative moments for the left section, AB. At the centre hinge B, the moment reduces to zero, as it should.

It will be found that this graphical representation of the moments to which an arch may be subjected is of value in determining the stresses in arches differently conditioned, such as arches with fixed ends or arches with two hinges.

Art. 4.—Influence Lines for Reactions in Three-hinged Arches.

Let it be required to determine the variation of the horizontal and vertical components of the reactions at A and C as any load P passes over the three-hinged arch shown in Fig. 7.

The horizontal projection of the two parts of the arch are represented by l_1 and l_2, while the total span is l. The rise of the centre hinge above the horizontal line A and C is h.

Taking B as the origin of coordinates, and measuring x, which is the distance in either direction of any load P,

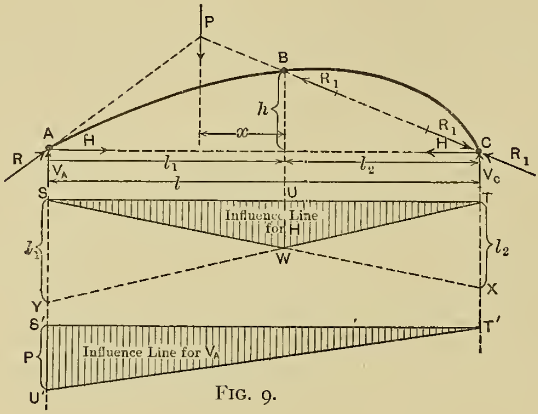


FIG. 7.

FIG. 8.

FIG. 9.

from B, the horizontal component H of the reaction at A, for the load between A and B, is

$$H = \frac{V_c \cdot l_2}{h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

V_c represents the vertical component of the reaction at C. But

$$V_c = \frac{P(l_1 - x)}{l},$$

and therefore

$$H = \frac{P(l_1 - x)l_2}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This expression may be represented graphically by a straight line, for the variable x appears in the first power only.

Erecting, then, in Fig. 8, on the base line ST a vertical TX below the hinge C equal to l_2 and connecting X with

S, it will be found that the portion of XS, viz., WS, included between vertical lines through A and B, will represent the influence line for H when the load lies between A and B. For by similar triangles the ordinate directly below the load is equal to $\frac{(l_1-x)l_2}{l}$. Hence the line WS is the influence line for a unit load.

In the same manner the line WT may be drawn to complete the diagram, the ordinate SY being erected below A equal to l_1 and the line YT then being drawn.

The final influence line for H is therefore SWT, and it obtains its maximum value for a load at the centre hinge, i.e.,

$$H_{\max}=\frac{P(l_1l_2)}{l}.$$

The influence lines for the vertical components V_a and V_c differ in no way from those found for simple beams, in consequence of the statical conditions of the structure.

In Fig. 9, therefore, the line $T'U'$ represents the influence line for V_a, and a similar line sloping in the opposite direction would be used for V_c.

It has been shown in Figs. 8 and 9 that each of the rectangular components of each reaction varies as the ordinates of a straight line; consequently their resultant, or the reaction itself, must vary in precisely the same manner. Hence, as the load P passes over the bridge, the variation in the value of the reaction R may also be represented by a straight line. The slope of such a line is a question of no importance; it suffices to know the law of variation.

Art. 5.—Influence Line for Stress in any Chord Member of a Three-hinged Arch.

Let Fig. 10 represent a three-hinged spandrel-braced arch with a horizontal upper chord and with vertical and inclined web members. The treatment of the problem, although applied to this particular truss, will be given in the most general manner so as to apply to any other form of three-hinged arch.

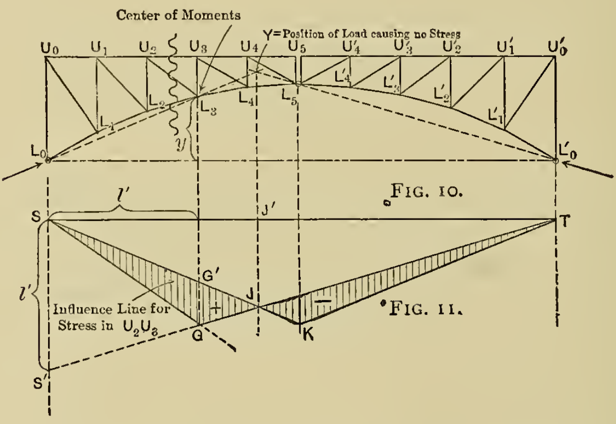


FIG. 10.

FIG. 11.

Let it be desired to find the influence line for stress in the chord member U_2U_3. Passing a section through the members U_2U_3, U_2L_3, and L_2L_3, and taking the centre of moments at L_3, the stress in U_2U_3 will evidently vary at precisely the same rate as the moment of the external forces taken about the point L_3. The influence line for stress in U_2U_3 may, therefore, be represented by the moment influence line for the point L_3. It has already been shown (Art. 3) that the moment at any point in a three-hinged arch is represented by the product of the pole distance by the vertical intercept between the centre of moments

on the arch and the true funicular polygon. For the truss shown, such a variable moment M for a single, unit load passing along the structure is represented by

$$M = M_v - H \cdot y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where M_v is the variable moment for the load passing over a simple beam with a span length L_0L_0', and where the product $H \cdot y$ is the moment due to the horizontal component of the reaction, y evidently being constant. On the base line ST (Fig. 11) the influence line for M_v may then be drawn in the usual manner, by erecting at S an ordinate SS', equal to the distance l' between the centre of moments L_3 and the left-hand abutment L_0, and connecting S' with T. Performing a similar operation for the right point of support, the influence line M_v is represented by SGT. From this line there is to be subtracted at every point the moment $H \cdot y$, which may be found in the following manner:

If the line of action of the reaction at L_0 passes through L_3, the centre of moments, and if the load causing this reaction should be placed to the right of the section passing through the members cut, the resultant moment about L_3 will be equal to zero; for the only force to the left of the section will be the reaction and its lever-arm about the centre of moments is zero. Such a position of the load causing no stress is found at the point Y, which is the intersection of lines drawn through L_0 and L_3, and L_0' and L_5; for the polygon L_0YL_0' is the funicular polygon for this one position of the load, since it passes through the three hinges. This position of the load, causing no stress in some particular member, will in future always be represented by Y. Projecting this point vertically downward until it intersects GT at J will fix that point on the final influence line, at which the stress is zero.

It has been shown in the previous article that the horizontal component of the reaction H varies as the ordinates of a triangle, the apex of the triangle being directly below the centre hinge. In eq. (1), under consideration, the product of H and y is required, but since y is constant the product will vary in the same manner as H; that is, as straight lines between the end and centre hinges. Three points of the variable $H \cdot y$ are known, viz., S, T, and J. The lines SJK and KT may then at once be drawn to represent the variation of $H \cdot y$, and any ordinate in the areas SGJ and JKT will represent the moment for the member U_2U_3 for the load placed over such ordinate; for, according to eq. (1), that ordinate expresses the relation $M_v - H \cdot y$. The shaded portion to the right of the point Y will represent negative moments, and the other shaded portion positive moments. For maximum stress of one kind the structure must then be covered with load from the point L_0' to Y, and for maximum stress of the opposite kind from L_0 to Y. The magnitude of the stress may at once be determined if the scale of the influence line is known; and this scale may be found if the stress in the member U_2U_3 be known for the load at any position, such as at U_3.

If, therefore, an ordinary force diagram be drawn for the entire structure for a single load P at any point, such as U_3, the stress in every member of the structure will be determined for this position of the load, and consequently also the scale of every influence line. In the present instance, for the member U_2U_3, the force diagram would furnish the true value of GG', and consequently the scale for any other ordinate in the area SGJ or JKT. It is then only necessary to multiply the area of the triangles SGJ and JKT by the proper intensity of loading to obtain the final maximum stresses for U_2U_3. This method of determining the scale of any influence line by means of a

force diagram drawn for a single load placed at any one fixed position will be applied to the case of web members, and it will not be necessary to repeat this explanation for those members.

The treatment of the stress in a chord member, such as $U_5 U_6$, Fig. 12, for which the centre of moments, L_5, is not vertically below a panel point, follows precisely the same methods; but since the influence line between the panel

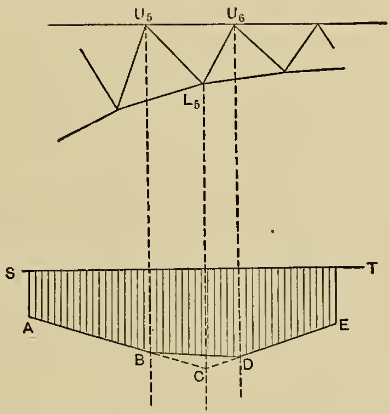


FIG. 12.

points U_5 and U_6 is a straight line, it becomes necessary to remove the angle BCD of the influence line ACE, precisely as in the case of the simple truss of Art. 11, Chap. II. The influence line ACE is, therefore, drawn considering a point vertically above L_5 as the centre of moments; the intersections of verticals drawn from the ends U_5 and U_6 of the panel to AC and CE determine the points B and D. Between B and D the influence line must be a straight line, and the final line is therefore $ABDE$.

Art. 6.—Influence Line for Stress in any Web Member of a Three-hinged Arch.

Let it be desired to find the influence line for stress in the member U_1L_2, Fig. 13, as a single load P passes along the structure. It may easily be shown that as long as the load P is to the left of the panel cut by the section passed through the members U_1U_2, U_1L_2, and L_1L_2, the

FIG. 13.

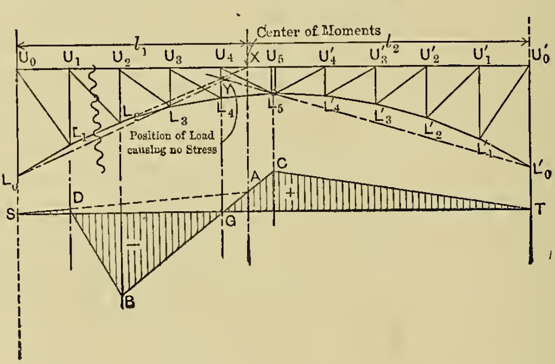


FIG. 14.

variation in the stress may be represented by a straight line. For the only external force to the right of the section is the right-hand reaction, which varies as a straight line for a single load, and as the moment of the reaction is taken about X, the intersection of U_1U_2 and L_1L_2, the variation in the stress of U_1L_2 for a load to the left of the section may also be represented by a straight line. Similarly, for the load on the right half of the arch between U_2 and U'_0, the stress varies directly as the left-hand reaction, and may be represented by straight lines intersecting in the vertical dropped from the hinge. This variation is similar to that of the left-hand reaction.

As the load passes over the panel U_1U_2, through which the section has been taken, the variation of the stress may be represented by a straight line, according to the treat-

ment of Art. 10, Chap. II. Four straight lines are, therefore, required in determining the influence line for stress in web members.

The stress in U_1L_2 is zero for a load placed at the point Y, this point being determined in precisely the same manner as in the case for chord members, i.e., it is the intersection of a line connecting the points L_0' and L_5 with the line connecting L_0 with X, the centre of moments. This point Y is projected vertically downward on the base line ST to G, and furnishes one point of the influence line.

It has been shown (Art. 16, Chap. II), in the case of the influence line for web members for simple non-continuous trusses, that the variation of the slopes of the positive and negative portions of those lines may be found by drawing lines from any point projected vertically below the centre of moments X to the ends of the span. This rule applies also to that half, U_0U_5, of the structure shown, of which the web member in question is a part, as the following analysis will show.

The origin of coordinates from which x, the position of the load P at any instant, is measured is at X, the centre of moments, and this point divides the span l into the two portions $l_1 = U_0X$ and $l_2 = XU_0'$. The vertical distance of X, the centre of moments, above the line connecting L_0 with L_0' is y and the corresponding distance of the centre hinge above this line is h. The vertical components of the left- and right-hand reactions at L_0 and L_0' are V_1 and V_2, and their horizontal component is H.

Then for a load of *unity* between U_0 and U_1, the moment of the stress S_1 for U_1L_2 becomes, by taking moments about X of the external forces to the right of the section,

$$S_1 \cdot g = V_2 l_2 - Hy,$$

g being the lever-arm of U_1L_2 about X.

But $$V_2=\frac{l_1-x}{l} \quad \text{and} \quad H=\frac{V_2 l_2}{h}.$$

Therefore $$S_1\cdot g=\frac{(l_1-x)l_2}{l}-\frac{(l_1-x)l_2\cdot y}{lh} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly, for a load unity between U_2 and U_5, the moment of the stress S_2 becomes

$$S_2\cdot g=V_1 l_1-Hy;$$

but $$V_1=\frac{(l_2+x)}{l} \quad \text{and} \quad H=\frac{V_1 l_1}{h}.$$

Therefore $$S_2\cdot g=\frac{(l_2\times x)l_1}{l}-\frac{(l_2+x)l_1\cdot y}{lh} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Both equations (1) and (2) represent straight lines, since the variable x is of the first degree only. When $x=0$,

$$S_1=S_2=\frac{l_1 l_2}{g\cdot l}\left(1-\frac{y}{h}\right);$$

that is, these two lines intersect in a point vertically below the origin of coordinates X.

In Fig. 14, therefore, A is any point below X, and the two lines AS and AB are drawn to represent portions of the influence line, for a load at S causes no stress in U_1L_2, nor, as just explained, does a load at G. Again, the lines passing through S and G must intersect on the vertical below X.

The remainder of the influence line may then be drawn from the conclusions reached in the beginning of this article. The line BA must be continued as a straight line to C vertically under the centre hinge, for the variation of stress in U_1L_2 is a straight line for the portion of the span U_2U_5; SD is the influence line for a load between U_0 and U_1; DB that for a load between U_1 and U_2, and CT that for the portion of the span U_5U_0'. The shaded areas

above the base line ST indicate stress of one kind and show not only what portion of the span must be covered with uniform load, but also the magnitude of the stress caused by that loading, for its value may be found by multiplying the positive shaded area by the intensity of the loading. As before the scale of the influence line may be found by drawing a stress diagram for the entire arch for one load at any point and comparing the value of the stress for this load with the ordinate below it on the influence line.

The shaded area below the base line indicates the portion of the structure to be covered by the load to cause negative stress, and its value may be calculated as just indicated.

Figs. 15 and 16 illustrate the influence line for stress

FIG. 15.

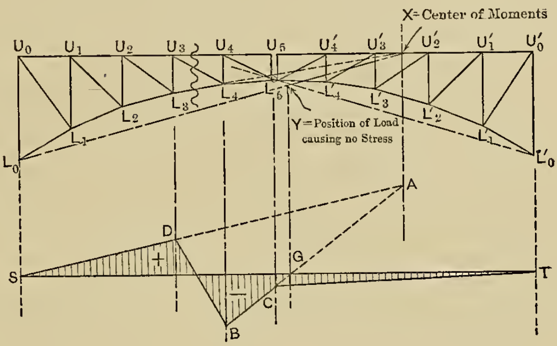


FIG. 16

in the web member U_3L_4 of the same three-hinged arch, but for this member the centre of moments X falls on the opposite side of the centre hinge, as does the point Y, the position of the load causing no stress. This influence line requires a slight additional explanation. As before, the influence lines for the portions U_0U_3 and U_4U_5 of the truss must intersect at A, and they must pass through S and G respectively, G being the projection of Y on ST. They are shown in Fig. 15 as SD and BG, but since the

influence line for the portion U_5U_0' is a *continuous* straight line, the point C where BG cuts the vertical through the centre hinge must be connected to T. The final influence line is, therefore, $SDBCT$, and it is at once evident what portions of the structure must be covered by the load to cause the maximum stresses of opposite kinds.

If the web members are alternately inclined, as in the form of webbing adopted in the Warren truss, the influence line suffers for that panel length of which the web member is a part, the modification explained in Art. 11, Chap. II; this change is also precisely the same applied to the chord members in Art. 5 of this chapter.

CHAPTER IV.

CANTILEVERS.

Art. 1.—Definitions.

A BEAM continuous over n points of support (Fig. 1) of which $n-1$ points are rolling supports, permitting longitudinal motion, requires the determination of n reactions. For the case of vertical loads only and under the condition that the rolling points of support move without friction on horizontal surfaces, two equations of condition only can be applied, namely, $\Sigma V=0$ and $\Sigma M=0$, since the third equation of condition, $\Sigma H=0$, disappears. There are required, therefore, for finding all the reactions $n-2$ other equations of condition, if the problem is to be statically determinate. Such equations are afforded by the insertion of $n-2$ hinges in the structure between the extreme points of support. Every hinge corresponds to an equation expressing the condition that the sum of the moments of the external forces on one side of it and about that point must be equal to zero. It should be observed, however, that without disturbing equilibrium two hinges only may be inserted between two adjacent points of support. A continuous beam may, therefore, be made statically determinate by the insertion of $n-2$ hinges. Such structures are termed cantilevers.

Fig. 1 illustrates a cantilever having four points of support and two hinges, at A and B. The span AB represents then a beam simply supported at its ends. The

spans CD and EF are termed anchor-arms and the portions DA and BE cantilever-arms. Other types of cantilevers may be constructed in which the hinges A and B occur in the spans CD and EF respectively. Such forms will not be analyzed separately, but the analysis deduced from Fig. 1 will be of such general character that it may be easily applied to any other form.

Art. 2.—Bending Moments in Cantilevers.

The funicular polygon is applied to a cantilever for the purpose of determining the moments at the various sections of the structure. Let the loads P_1, P_2, P_3 . . . P_{11} (Fig. 1) act on the various spans, as shown. Fig. 3 shows these loads laid down in regular order, with the poles O', O'', O''', chosen arbitrarily, but all with the same pole distance. One pole might have been chosen, but it will prove more convenient to choose separate poles for the loads on the different span lengths CD, DE, and EF. The funicular polygon (Fig. 2) is then constructed in the usual manner with the rays 1, 2 . . . 14, but no closing lines can as yet be determined. The positions of the closing lines may be found immediately, however, since directly below the points A and B there are points of zero bending. The bar 16 is, therefore, drawn in Fig. 2 so as to show zero moments below A and B. At the points where bar 16 intersects the lines of the reactions R_2 and R_3, bars 15 and 17 may then be drawn to the reactions R_1 and R_4 respectively, showing zero moments below points C and F. The shaded portions of the funicular polygon (Fig. 2) represent, then, in the usual manner the bending moments at the various sections of the structure for the fixed position of the loading shown.

The magnitudes of the reactions R_1, R_2, R_3, and R_4

may also be found from Fig. 3, it being only necessary to insert the three closing rays 15, 16, and 17 through

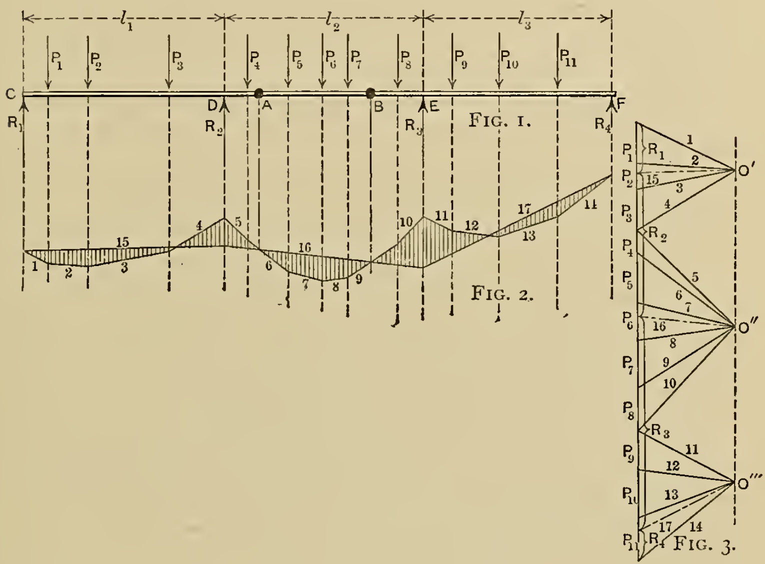


FIG. 1.
FIG. 2.
FIG. 3.

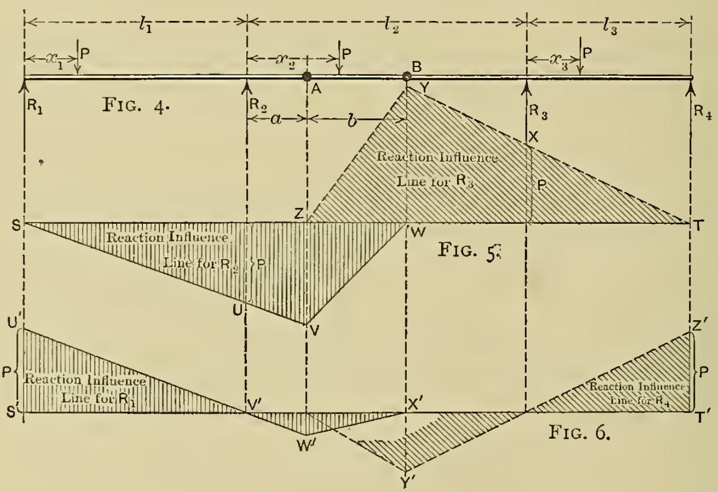
the poles O', O'', and O''' respectively. The brackets enclosing portions of the load line furnish the desired quantities.

Art. 3.—Reaction Influence Lines for Cantilevers.

Fig. 4 illustrates a general type of cantilever structure in which the hinges are at A and B, and for which it is desired to represent the variations of the reactions R_1, R_2, R_3, and R_4 for any load P passing over the structure. Let the influence line for R_2 be first determined. If the load P be placed at R_2 the reaction at that point must be equal to P and an ordinate may be erected (Fig. 5) below that point on the base line ST, equal to P. It is evident that for a load between R_1 and R_2 the reaction R_2 varies

directly as the distance from R_1; that is, it is equal to $P\frac{x_1}{l_1}$ and is graphically represented by the line SU. For any load between R_2 and the hinge at A, the variation of R_2 will be indicated by a continuation of the line SU to the point V vertically below A, since R_2 still remains equal to $P\frac{x_1}{l_1}$.

A load P placed at the point B fails to affect the reaction R_2, being carried entirely to R_3 and R_4. Between B and



A, R_2 varies directly as the ordinates of a straight line, for precisely the same reason that all influence lines are straight lines between panel points; since the points W and V of this line have been determined, this portion of the influence line is represented by VW. The ordinates between ST and $SUVW$ therefore represent the variations of R_2 as any load passes over the structure. It is evident that no load between the points B and R_4 influence the reaction R_2.

In an exactly similar manner the influence line for R_3 may be represented by the line $TXYZ$, the slope of the line TXY being found by erecting at R_3 an ordinate of height P.

The influence line for the reaction at R_1 is represented in Fig. 6. For a load at R_1 the reaction is P and is represented by the ordinate $S'U'$ erected at that point. As the load P passes from R_1 to R_2, R_1 varies as the ordinates of a straight line, the value of R_1 being zero when P is at R_2. This rate of variation remains constant as far as the point A, it being noted, however, that after passing R_2 the reaction R_1 becomes negative. This is indicated by the diagram, where the ordinates between the base line $S'T'$ and the line $V'W'$ are drawn below the base line. The variation in the reaction R_1 for loads on AB is represented graphically by the line $W'X'$, and is found in precisely the same manner as for R_2 and R_3.

As a general conclusion, it is clear that for the greatest upward value of R_1, the distance $S'V'$ must be covered with load, and for the greatest downward reaction or uplift the distance $V'X'$ must be covered. Loads on other portions of the structure do not affect the reaction at R_1. The influence line for R_4 is also represented in Fig. 6, but requires no detailed explanation.

Inspection of Figs. 5 and 6 will show that the slopes of the pairs of lines SUV and $U'V'W'$, and YXT and $Y'Z'$, are respectively the same. This must evidently be the case, for at any point in the spans represented by these lines the algebraic sum of the pairs of reactions, viz., of R_1 and R_2, and of R_3 and R_4, must always equal P. It is to be noted that the distance AB, or the suspended span, is not included in these lengths.

Art. 4.—Shear Influence Lines for Cantilevers.

The shear influence lines may at once be drawn by means of the reaction influence lines. Let the shear in the section MN situated between R_1 and R_2 (Fig. 7) be required, that figure representing a part of a cantilever structure resting on the supports R_1, R_2, R_3 . . . The base line ST (Fig. 8) and the reaction influence line for R_1,

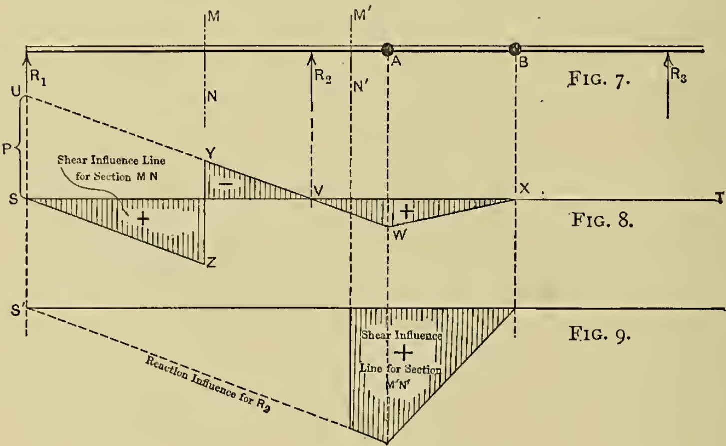


FIG. 7.

FIG. 8.

FIG. 9.

$UVWX$, are first drawn. For any load between the hinge B and the section MN the shear for MN is always equal to the reaction R_1, but between MN and R_1 the reaction must be diminished by the amount of the load P, as shown in the diagram. The resulting shear influence line for the section MN is then the line $XWVYZS$. It is clear then that for the greatest positive shear at MN the span R_1R_2 must be covered by the load extending from R_1 to the section, while the cantilever-arm R_2A and the suspended span AB are also covered. There should be no load between the section MN and R_2. For the greatest

negative shear, however, the load should be placed only between MN and R_2.

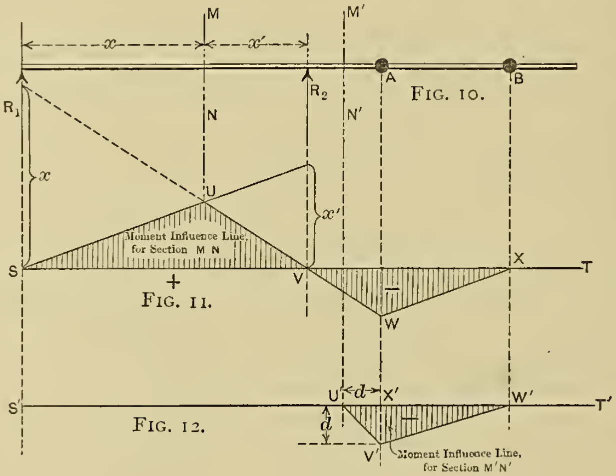
In a precisely similar way the shear influence line for the section $M'N'$, at any point in the cantilever-arm R_2A, may be drawn. The reaction influence line for R_2 (Fig. 9) is drawn and that portion of it included between the section $M'N'$ and the hinge B represents the shear influence line for the section $M'N'$. The shaded portion of the diagram represents the sum of all the positive shears. It is evident from this diagram that positive shear only can exist at $M'N'$ and for the maximum value the suspended span and also that portion of the cantilever span included between the section $M'N'$ in question and the hinge A should be covered with the load. The load on other portions of the structure does not affect in any way the shear in the cantilever-arm.

These two cases suffice to illustrate the variations of shear in any cantilever structure. If the latter have parallel and horizontal chords, the maximum web stresses may at once be obtained by multiplying the maximum shears by the secants of the inclinations of the web members from a vertical. The treatment of influence areas is at once applicable to cantilevers, for the span lengths being usually large, a uniform loading may replace the locomotive concentrations without essential error. Locomotive concentrations require the use of a criterion for the exact determinations of stresses.

The division of the structure into panel lengths has not been treated in detail for shears, nor will it be for moments, for it is now well established that the effect of a panel on a general influence line is simply to remove all angles from it below the panel.

Art. 5.—Moment Influence Lines for Cantilevers.

Fig. 10 represents partially a cantilever structure in which it is desired to obtain the influence line for moments at the section MN in the anchor-arm R_1R_2. For loads between R_1 and R_2 it is evident that the moment influence



line for MN is exactly the same as that for a simple non-continuous structure. It is only necessary to construct ordinates equal to x and x' on the base line ST (Fig. 11) at the points S and V below the reactions R_1 and R_2 respectively, and connect the ends of these lines to V and S. The influence line for that portion of the span is then the line SUV.

The moment at the section MN for any load on the cantilever-arm R_2A is represented by R_1x. It is seen that this quantity varies at the same rate as R_1, since x is a constant in this expression; R_1x may therefore be represented by a continuation of the line UV to W. Since R_1 is negative for this portion of the span, the

moments are also negative, as represented by the influence line. As the load at B causes no bending at MN the variation of the moment for a load passing along AB is represented by the line WX. The final influence line is therefore $SUVWX$, the portion SUV representing positive moments and the portion VWX negative moments. Positive moments may be regarded as moments causing compression in the upper chord members and tension in the lower chord.

The influence line for moments at any section $M'N'$ in the cantilever-arm is represented by Fig. 12. For a load between $M'N'$ and A the moment varies directly as its distance from $M'N'$, and it is only necessary to erect below A an ordinate $X'V'$ equal to d or the distance between $M'N'$ and A, and to draw the line $U'V'$; this line then represents the variation in the moment at $M'N'$, for the distance shown. For loads on the suspended span AB the variation in the moment is then represented by $V'W'$, and the completed influence line is $U'V'W'$.

It is thus shown that all moments in the cantilever-arm are negative or such as to cause stresses of tension in the upper chord members, and that for their maximum values the entire suspended span and that portion of the cantilever-arm between $M'N'$ and A must be covered with load. Loads on other portions of the structure in no way affect the moment at $M'N'$.

Stress influence lines for the chord members of cantilevers are not required, since the moment influence lines also suffice to furnish the stresses in those members by dividing by the proper lever-arms. It is only necessary to measure the various shaded areas shown in the figures and to multiply them by the proper intensity of the uniform load and then to divide by the lever-arm. To determine the scales of the various figures, it is sufficient to place a unit load at any convenient point and obtain the desired

moment for that position. This determines the value or the scale of the ordinate at that point, and therefore the scale of the entire figure.

Art. 6.—Stress Influence Lines for Members of Cantilevers.

Fig. 13 illustrates a portion of a cantilever structure in which the span U_0U_5 is the anchor-arm, U_5U_9 the cantilever-arm, and U_9U_{16} the suspended span. Other portions of the structure to the right of U_{16} do not affect the stresses in any of the members shown in Fig. 13.

Fig. 14 illustrates the influence line for stress in the member U_6L_7 of the cantilever-arm. Passing a section through U_6U_7, U_6L_7, and L_6L_7, the centre of moments falls at a, projected vertically to the base line MN at C. As a load passes from U_7 to U_9 the stress in U_6L_7 varies directly as the distance of that wheel load from a or C, for the stress in U_6L_7 equals the moment about that point divided by the lever-arm ab, and it may be represented graphically by BD, a portion of the line BC. A load at either U_6 or U_{16} causes no stress in U_6L_7. The final line, therefore, is $ABDN$.

Fig. 15 shows the influence line for web member U_2L_3 of the anchor-arm; for a load on the span U_0U_5 the stress may be represented by the influence line $FGHIJ$, precisely as for a non-continuous simple span. The line IJ is then continued to K, the vertical projection of U_9, and then connected to N. The final line is the line bounding the shaded area shown in the figure.

Figs. 16 and 17 show the influence lines for the members L_8U_9 and U_2U_3 respectively. They require no detailed explanation, for they represent the variation of the bending moments at the points U_9 and U_3 respectively.

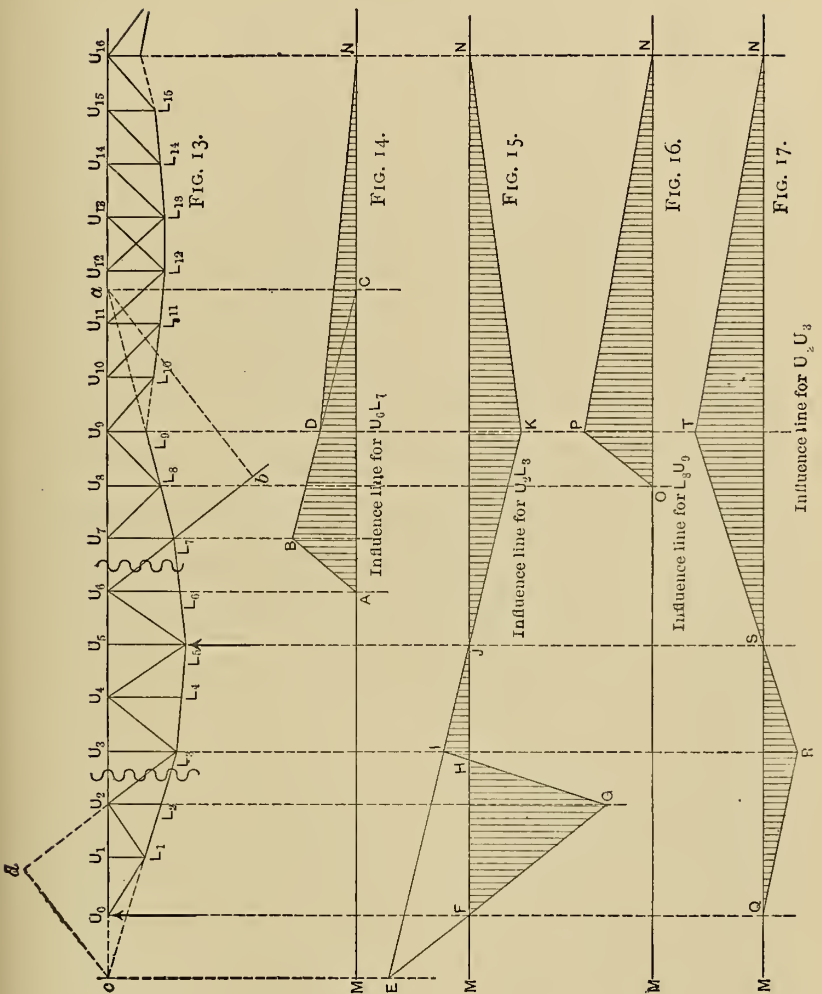


FIG. 13.

FIG. 14.

FIG. 15.

FIG. 16.

FIG. 17.

Fig. 18 represents a complete cantilever structure, for which U_0U_6 and $U_{19}U_{25}$ are the suspended spans, U_6U_9 and $U_{16}U_{19}$ the cantilever-arms, and U_9U_{16} the anchor-span.

Fig. 19 exhibits the influence line for stress in $U_{11}L_{12}$; the portion $BCDE$ is found by treating the load on the anchor-span as if the latter were a simple non-continuous structure. The centre of moments falls at b and is projected vertically downward to F, from which point the lines BC and DE are drawn through points directly below the ends of the span U_9U_{16}. BC is then continued to A and DE to G, the ends of the cantilever-arm. These points are connected to M and N. The final line is, therefore, $MABCDEFGN$, and shows the position of loading for both maximum and minimum stress, and also the stress area.

Fig. 20 is drawn in an exactly similar way for the member $U_{10}L_{11}$, but in this case the centre of moments falls at a, within the anchor-span, and the influence line suffers the changes shown.

Fig. 21 is the influence line for stress in the chord member $U_{11}U_{12}$, the centre of moments for which is projected down to S.

Cantilever on Towers.

A constructive device used advantageously in cases where the cantilevers are supported on high iron or steel piers or towers is shown in Fig. 22. It leads to appreciable economy by the shortening of the clear cantilever opening as well as the anchor-arm. It consists in separating the feet of the two inclined posts L_7L_8 and L_9L_{10} to some convenient distance and omitting all bracing in the rectangle $U_8U_9L_8L_9$. No shear can then be transferred past L_9 to the left or past L_8 to the right. Since this last condition exists, the reaction R_1 at L_0, and the positions of loading

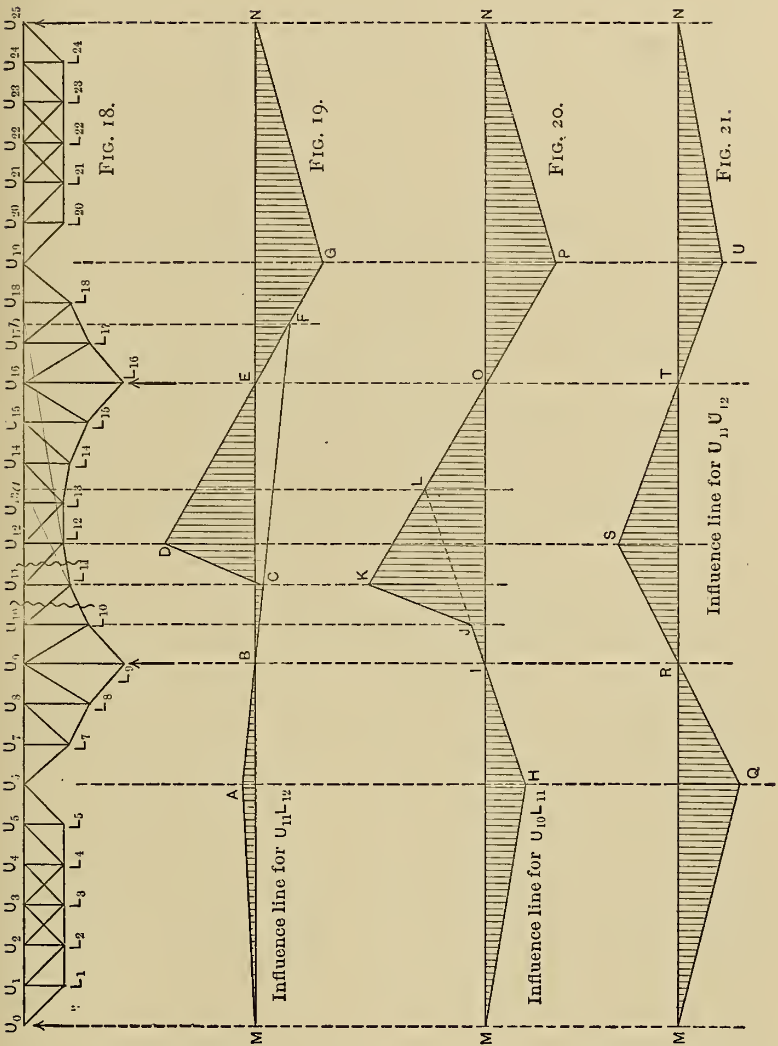


FIG. 18.

FIG. 19.

Influence line for $U_{11}L_{12}$

FIG. 20.

Influence line for $U_{10}L_{11}$

FIG. 21.

Influence line for $U_{11}U_{12}$

for all the maximum web and chord stresses in the cantilever structure, as found from the preceding constructions, will hold for this case without any change whatever. It is only necessary to observe carefully that the open panel U_8U_9 is to be treated as reduced to a point or entirely neglected. This is shown in Fig. 26, which is the influence line for

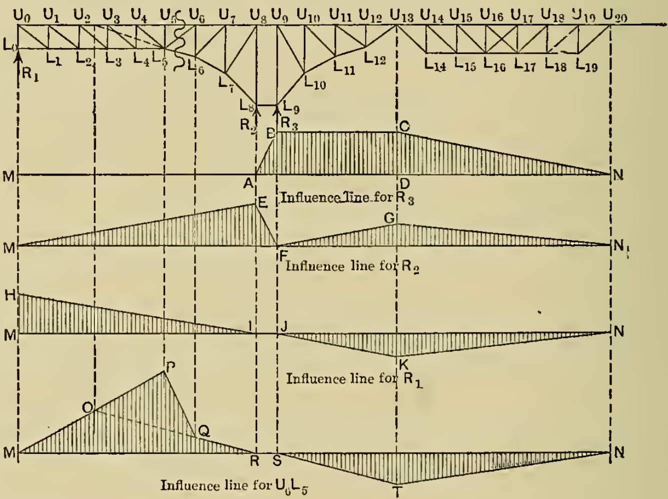
FIG. 22.

FIG. 23.

FIG. 24.

FIG. 25.

FIG. 26.



U_6L_5. The line ST is parallel to QR, and would form a continuation of it if the point S were transferred to R.

The only changes necessary are for the reactions R_2 and R_3, at L_8 and L_9 respectively, and the determination of the stresses in the members U_8U_9 and L_8L_9. These latter stresses are always equal to each other, but with opposite signs; they may be found by obtaining the maximum moment at L_9 and dividing by the height of tower, U_9L_9. The reaction influence lines for R_3 and R_2 are shown in Figs. 23 and 24.

In the case of Fig. 23, the ordinate below U_9 is equal to the load itself, or P, and diminishes as a straight line BA to U_8. R_3 remains equal to P for the load between U_9

and the end of the cantilever at U_{13}, but diminishes as shown by CN from that point to U_{20}, being always equal to the reaction of the simple suspended span at U_{13}.

Fig. 24 exhibits the variation of the reaction at R_2. It is equal to P when the latter is at L_8, but it equals zero for the load at U_0, U_9, and U_{20}. The slope of the line FG is a continuation of the slope of ME, i.e., FG is parallel to ME. The final influence area is that shown shaded. It is seen that no uplift ever occurs at R_2.

The variation of R_1 as the load P passes over the structure is shown by Fig. 25, and it differs in no way from that found without the open panel U_8U_9. The line HI is parallel to JK.

As a check on the construction of these three figures, it is only necessary to note that at any one point the algebraic sum of the ordinates for R_1, R_2, and R_3 must equal P, the load. The diagrams fulfil this condition.

Art. 7.—Suspended Cantilevers.

The structure shown in Fig. 27 is a true cantilever, although it possesses some of the lines of a suspension bridge, and the term *suspended cantilever* may be applied

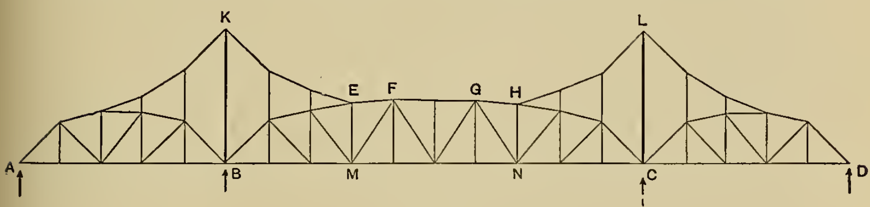


Fig. 27.

to it. Some general deductions as to its statical condition, and the application of the method of influence lines to determine the maximum stresses, will be found in the following article:

Statical Condition of the Structure.

Fig. 27 shows a coplanar frame work resting on the four points of support A, B, C, and D, and any one of them may be considered fixed while all the others are supported on nests of rollers moving on horizontal beds. These conditions of support eliminate temperature stresses. The structure is then virtually a stiffened suspension bridge with vertical anchorages, the cable being $AKEFGHLD$, while the road bed lies along the lower chord $ABCD$.

If the structure be inverted and supported at the points $AKLD$ of which only one is, as before, fixed, the framework becomes a continuous bridge of three spans, not subjected to temperature stresses; it may be considered a stiffened arch with vertical reactions. If the structure is supported at K and L as fixed points it becomes a two-hinged arch subjected to temperature stresses.

In all these cases, even where the reactions are vertical, there are four such reactions to be found; therefore, Fig. 27 is a statically indeterminate framework; that is, the three equations of condition of coplanar statics are insufficient for the determination of the stresses in the members. There are required, in addition, other equations of condition, depending either on the elasticity of the material or the work performed in the structure by the displacement of its members.

The structure, shown in Fig. 27 may, however, be made statically determinate by the removal of the members EF and GH, as shown in Fig. 28, and by assuming the two panel points M and N to be frictionless pins. The structure is then a suspension cantilever in which the spans AB and CD are the anchor arms, the spans BM and NC the cantilever arms and MN is the suspended span. No difficulty will be experienced in determining for this class of structure

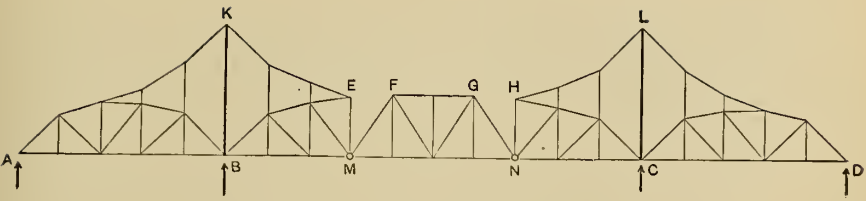


FIG. 28.

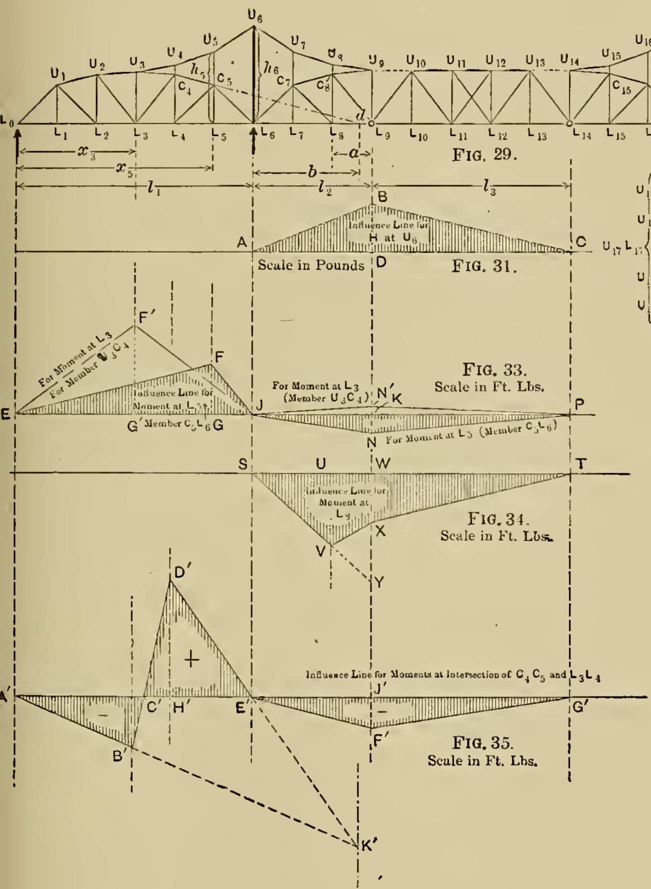


FIG. 29.

FIG. 31.

FIG. 33.
Scale in Ft. Lbs.

FIG. 34.
Scale in Ft. Lbs.

FIG. 35.
Scale in Ft. Lbs.

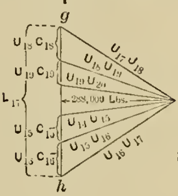


FIG. 32.
Scale in Pounds

the stresses due to dead load. The method of influence lines is, however, particularly applicable for determining the positions of the moving load causing the maximum stresses, as well as for determining the maximum stresses themselves.

Fig. 29 represents a suspension cantilever, the span $l_1 = L_0L_6$ being the anchor arm, the span $l_2 = L_6L_9$ being the cantilever arm and the span $l_3 = L_9L_{14}$ being the suspended span. The entire structure is taken to be symmetrical about the center of the panel $L_{11}L_{12}$. The distances measured between the lower panel points L and the upper panel points U are designated by h with a proper subscript; for instance, U_5L_5 is h_5 and U_6L_6 is h_6.

It will be assumed that the points L_0 and L_{23} are anchored and capable of carrying negative or downward reactions; the points L_6 and L_{17} are supported on nests of rollers moving on horizontal beds. For the static determination of the stresses and for the elimination of temperature stresses it also becomes necessary that one of the points of support of the suspended span be so carried that it may move horizontally over the end of the supporting cantilever arm; the point L_{14} of the suspended span may, for instance, be placed in a slotted hole.

Art. 8.—Fixed Load Stresses.

Fig. 30 shows a portion of the structure of Fig. 29 for which it is desired to obtain the stresses due to fixed load. For case of illustration, the dead loads $W_1W_2W_3 \ldots W_9$ are all assumed to be supported at the lower panel points only. Their actual distribution between upper and lower chords would add, however, no difficulty to the work.

The reaction R_0 at L_0 may be obtained at once by taking the moments of all external forces about L_6 and placing

them equal to zero. It is to be observed that the weight W_9 includes one half the weight of the suspended span. The equation involving the reaction R_0 as the unknown quantity will then have the form

$$R\ l_1 = \pm \Sigma Wx$$

where x represents the distance of any panel weight from L_6 and is to be taken plus or minus as it is measured to the right or left of L_6.

The usual force polygon may then be constructed, taking at first the unknown forces L_0U_1 and $L\ L_1$ acting at panel point L_0 and balancing the reaction R_0. In order to simplify the terminology, the members will for the

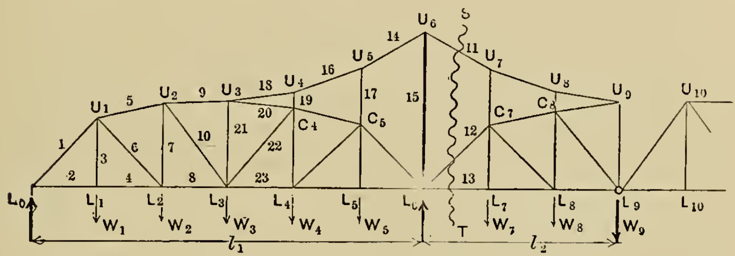


FIG. 30.

present also be designated by the numbers placed upon them. Having found, therefore, the stresses in the members 1 and 2, the stresses in the members 3 to 10 inclusive, taken in numerical order, may then be determined. At panel point U_3 there are, however, three unknown concurrent forces, and one of them must be known before the others can be found by means of the force polygon.

If a section ST be passed through the members 11, 12 and 13, the stress in the member 11 may be found by taking moments of all the external forces situated to the right of the section (namely, W_7, W_8 and W_9) about L_6 as the center

of moments. The stress in member 11, multiplied by its lever arm about L_6 is then equal to the sum of the moments of these external forces. The stress in member 11 having thus been obtained, the stresses in members 14 and 15 may at once be determined, for they are the only two unknown forces acting at U_6. Panel point U_5 may then be treated in turn, and the stresses in the members 16 and 17 found. In precisely the same way there may then also be determined the stresses in the members 18 and 19.

One of the three unknown stresses at panel point U_3 has thus been found, and the determination of the stresses in members 20 and 21, or in any other members of the structure, presents no further difficulty.

Art. 9. Live Load Stresses.

In treating the live load stresses in the various spans of Fig. 29 it is first to be noted that the stresses in the suspended span l_3 are precisely the same as if that structure were a simple non-continuous span; it requires, therefore, no further consideration.

The anchor span l_1 is also a simple non-continuous span for any loads placed on that span only, the members U_3U_4, U_4U_5, U_4C_4, etc., not acting. Loads on the spans l_2 and l_3, however, cause stresses in the members of this span and these stresses must be investigated. Since they are caused by the stresses in the members U_3U_4, U_4U_5, and U_5U_6, and the suspender rods U_4C_4 and U_5C_5, these stresses must first be found. The horizontal components of all the upper chord members, such as U_4U_5, must necessarily be equal, since the only other members which are attached to them are vertical and carry no horizontal components. An influence line for the variation of this horizontal component as a load passes between the points L_6 and L_{17} will therefore be required.

Let there be placed a load P at L_9, the end of the cantilever arm l_2, and let a section be passed through the members U_6U_7, L_6C_7 and L_6L_7. The horizontal component of the stress in U_6U_7 may then be found by taking its moment and those of the external forces about L_6. The vertical component of U_6U_7, disappears in the moment equation since its lever arm about the center of moments is zero. If H represents the horizontal component, the following equation will result:

$$H \cdot h_6 = P \cdot l_2, \qquad (1)$$

or

$$H = \frac{P \cdot l_2}{h_6}. \qquad (2)$$

Equation 2 shows that the value of H varies directly as the distance of the load P from L_6; this variation may therefore be indicated by a straight line for the span l_2. The influence line for H may therefore be drawn as follows (Fig. 31):

At the point D, situated on the base line AC directly below L_9 the ordinate BD is erected with a value $H = P \cdot l_2/h_6$, and the point B is then connected to A by a straight line.

Any load on the span l_3 is divided by the principle of the lever into two component parts, acting at L_9 and L_{14}; that part at L_{14} causes no stress whatever in that portion of the structure to the left of L_9. That part of the load carried to L_9 varies directly as the distance of that load from L_9 and its effect on the influence line H may be represented by a straight line between L_9 and L_{14}, with a zero value at L_{14}. In Fig. 31 the influence line is therefore completed by connecting the point B to the point C. It is to be noted that this distribution of the load on the span l_3 causes every influence line between L_9 and L_{14} to be a straight line. Hereafter, therefore, if the value of the

ordinate below L_9 is known, its end may at once be connected to a point on the base line directly below L_{14}.

It is seen that every load on the spans l_2 and l_3 increases the value of H; the maximum value of H is therefore found by covering the spans l_2 and l_3 completely; if the load be taken as a uniform load (which it is proper to do for long spans) the area of the triangle ABC multiplied by the intensity of the uniform load will furnish the maximum value of H.

Let the following data apply to this bridge:

$U_1L_1 = 30$ feet $= h_1$,
$U_2L_2 = 38$ " $= h_2$,
$U_3L_3 = 40$ " $= U_9L_9 = U_{10}L_{10} = U_{11}L_{11} = h_3 = h_9$, etc.,
$U_4L_4 = 45$ " $= U_8L_8 = h_4 = h_8$,
$C_4L_4 = 38$ " $= C_8L_8$,
$U_5L_5 = 55$ " $= U_7L_7 = h_5 = h_7$,
$C_5L_5 = 30$ " $= C_7L_7$,
$U_6L_6 = 75$ " $= h_6$.

All panel lengths are taken equal at 30 feet each; the span l_1 is therefore 180 feet, the span l_2 90 feet, and the span l_3 150 feet.

The value of H for a load of unity at L_9 is therefore $\frac{90}{75} = 1.2$ and the ordinate BD is drawn with that height. The area of the triangle ABC is then $\frac{240 \times 1.2}{2} = 144$, and if the intensity of the loading be taken at 2000 pounds per linear foot of truss, the maximum value of H becomes 288,000 pounds. The maximum stresses in the vertical suspender rods and upper chord members U_3U_4, U_4U_5, etc., will then be found by means of the simple force diagram shown in Fig. 32. For clearness of illustration the right hand tower $U_{17}L_{17}$ instead of the left-hand tower is treated.

The horizontal force H is first laid down to proper scale

and from one end of it are drawn lines parallel to $U_{17}U_{18}$ and $U_{16}U_{17}$ until these lines intersect the vertical gh erected at the left end of H. These lines, when measured to scale, furnish the maximum stresses in those members. The stress in the tower column $U_{17}L_{17}$, is found in the same force triangle; its value is the distance gh or the vertical intercepted by the lines just drawn.

The maximum stresses in the suspender rods and upper chord members are found in the same figure by drawing lines respectively parallel to $U_{18}U_{19}$, $U_{19}U_{20}$, $U_{14}U_{15}$, etc. The stresses found for these members are clearly indicated in the figure. They are found from the simple principle of the force polygon as applied to the equilibrium of concurrent forces for at any point, such as U_{16}, there are to be determined only two unknown forces $U_{15}U_{16}$ and $U_{16}C_{16}$.

Art. 10.—Influence Lines for the Chord Members of the Anchor Span, L_0L_6.

The influence line for the member C_5L_6 of the span l_1 will first be drawn. It has already been shown that, for loads on the span l_1, the members of the anchor span are to be treated as if that span were a simple non-continuous span. For that condition, then, the stress in the member C_5L_6 is found by passing a section through the members U_5U_6, C_5L_6, and L_5L_6. Since the member U_5U_6 is not stressed, the center of moments may be taken at L_5.

Since the stress in C_5L_6 is equal to the moment at L_5 divided by a lever arm, its variation may be represented by the variation in the bending moment at L_5, and Fig. 33 represents the influence line for the moment at L_5. For the span l_1 this line is EFJ; the value of the ordinate FG is found by placing a unit load (1 pound) at L_5; the reaction at L_0 is then $R_0 = \frac{1}{6}$ and its moment about L_5 is $\frac{1}{6} \times 150 = 25$ foot-pounds or FG.

When loads are placed on the pans l_2 and l_3 the member U_5U_6 is stressed; but in taking moments about L_5 for the stress in the member C_5L_6 it will only be necessary to consider the horizontal component of the stress in U_5U_6, for the moment of its vertical component disappears, its lever arm being zero.

Let there be placed a load P at L_9 and let it be required to find the moment at L_5 for that position of the load. The reaction at L_0 is upward and is equal to

$$R_0 = \frac{P \cdot l_2}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If H represents the horizontal component of the stress in any upper chord member, the bending moment M_5 at L_5 is then given by the following equation:

$$M_5 = R_0 \cdot x_5 - H \cdot h_5. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substituting the values of R_0 and H from Eqs. (3) and (2),

$$M_5 = P\left(\frac{l_2 \cdot x_5}{l_1} - \frac{l_2 \cdot h_5}{h_6}\right). \quad . \quad . \quad . \quad . \quad (5)$$

Substituting the numerical values of these quantities, and assuming P to be a unit load,

$$M_5 = \left(\frac{90 \times 150}{180} - \frac{90 \times 55}{75}\right) = 9 \text{ foot-pounds.}$$

The ordinate KN below L_9 is drawn downwards with this value, for it causes a tensile stress in C_5L_6.

It is to be noted that the value of M_5 as given by Eq. (5) varies directly as l_2, or the distance of the load from L_6. The influence line (Fig. 33) for the spans l_2 and l_3 may therefore be completed by drawing straight lines from N to

J and P. The shaded areas show the portions of the structure to be covered by loading, to cause the maximum stresses of opposite kinds; it is to be remembered, however, that these areas are moment areas, and are to be divided by the lever arm of C_5L_6 in order to furnish stresses.

The value of the maximum moment, causing compression in C_5L_6 is

$$\text{area } EFJ \times 2000 = \frac{180 \times 25 \times 2000}{2} = 4,800,000 \text{ foot-pounds,}$$

while that causing tension is

$$\text{area } JNP \times 2000 = \frac{9 \times 240 \times 2000}{2} = 2,160,000 \text{ foot-pounds.}$$

The influence line lor the member U_3C_4 is shown in the same figure as $EF'J$ for the span l_1 and as $JN'P$ for the spans l_2 and l_3. For the latter spans, the stress is compressive just as for the span l_1, as the evaluation of Eq. (5) shows. Moments are taken about the panel point L_3, and there is found, for a load of 1 pound at L_9:

$$M_3 = \frac{90 \times 90}{180} - \frac{90 \times 40}{75} = -3 \text{ foot-pounds.}$$

The ordinate $G'F'$ is found by placing the unit load at L_3; the reaction R_0 becomes $\frac{1}{2}$, and the moment $\frac{1}{2} \times 90 = 45$ foot-pounds.

The influence line for a chord member, such as U_1U_2 is found in precisely the same manner by means of the same equations; in Eq. (5) however the term involving H disappears, and there is to be used

$$M_1 = P\frac{l_2 \cdot x_1}{l_1}. \qquad \qquad (5a)$$

Art. 11.—Influence Lines for the Chord Members of the Cantilever Arm L_6L_9.

The influence line for the member C_7C_8 will be drawn; it is at once evident that loads on the span l_1 cause no stress in any of the members of the spans l_2 or l_3, and that span need not be considered.

The stress in C_7C_8 is found by passing a section through U_7U_8, C_7C_8, C_7L_8, and L_7L_8, and then taking moments about L_8. The stress in U_7U_8 may be found, for any position of the load; its horizontal conponent need only be considered, for the moment of the vertical component disappears as its lever arm is zero.

If a load P be placed at L_8, and if M_8 represent the moment of the stress in C_7C_8 about L_8;

$$M_8 = H \cdot h_8, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

But

$$H = \frac{P(l_2 - a)}{h_6};$$

therefore

$$M_8 = P \cdot \frac{(l_2 - a)h_8}{h_6}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The stress in C_7C_8 will result at once from dividing M_8 by the normal distance from L_8 to that member.

For any position of the load between L_6 and L_8, Eq. (7) shows that M_8 varies directly as the first power of the variable distance $(l-a)$; this variation may therefore be represented by the straight line SV in Fig. 34, the ordinate VU directly below L_8 having the value furnished by Eq. (7).

If a load be placed at L_9, the moment becomes

$$M_8 = H \cdot h_8 - P \cdot a. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The value of the ordinate XW of the influence line erected at L_9 is therefore found by continuing the line SV to intersect the vertical dropped from L_9 and subtracting from it $P \cdot a$.

The influence line is then finally completed by connecting X and T by a straight line.

Substituting the numerical quantities in Eq. (7) and assuming P to be 1 pound, there is found,

$$M_8 = \frac{(90 - 30)45}{75} = 36 \text{ foot-pounds} = UV.$$

The value of XY is $P \cdot a = 30$ foot-pounds.

The shaded area of Fig. 34 when multiplied by the intensity of the uniform load, furnishes the maximum bending moment at L_8. When divided by the lever arm there will be obtained the maximum stress, which is compressive.

Art. 12.—Influence Line for Web Members of the Anchor Span L_0L_6.

The influence line for the member C_4L_4 will be drawn; its stress is found by passing a section through U_4U_5, C_4C_5, C_4L_4 and L_3L_4, and dividing the bending moment found at the intersection d of C_5C_4 and L_3L_4, distant b from L_6, by the lever arm of C_4L_4. It will then only be necessary to draw the influence line for moment at that point; the stress in C_4L_4 may be found directly from that moment.

Load on Span l_1.

The influence line for the load at any point of the span l_1, is found just as if that span were a simple truss; lines are drawn (Fig. 35) from any point K', found at any point directly below the center of moments, to points on the base line A' and E', directly below the ends of the simple span. The lines $A'B'$ and $D'E'$ are then portions of the influence line. In the panel L_3L_4, the influence line is also a straight line, and the line is finally completed by connecting B' and D' by a straight line.

If a load P be placed at L_4, the reaction at L_0 becomes in the present case $R_0 = \frac{1}{3}P$; and the moment of the external forces situated to the left of the section passed, is

$$M_d = \frac{P}{3}(l_1 + b) = P\left(\frac{180 + 82.5}{3}\right) = 87.5\,P.$$

If P be taken equal to 1 pound, the value of the ordinate $D'H'$ (Fig. 35) is therefore 87.5 foot-pounds. This value determines the scale of this portion of the influence line.

Load on Spans l_2 and l_3.

For a load placed on the spans l_2 or l_3, it will be necessary to obtain the stress in the member U_4U_5, for that stress appears as an unknown quantity in the equation of moments about the center of moments d.

It will be more convenient, however, to treat the horizontal and vertical components, H and V, of that stress.

Placing a load P at L_9, the moment M_d at d, then becomes:

$$M_d = R_0(l_1 + b) - Hh_4 - V(b + l_1 - x_4), \quad . \quad . \quad . \quad (9)$$

but

$$R_0 = P\frac{l_2}{l_1}, \qquad H = P\cdot\frac{l_2}{h_6}, \qquad \text{and} \qquad V = H\tan\alpha,$$

where α is the angle between the member U_4U_5 and a horizontal. Since $\tan \alpha = \frac{(h_5 - h_4)}{p}$, where p = panel length,

$$V = P \cdot \frac{l_2}{h_6} \frac{(h_5 - h_4)}{p}.$$

Substituting these quantities in Eq. (9),

$$M_d = P\left[\frac{l_2}{l_1}(l_1 + b) - \frac{l_2}{h_6}h_4 - (b + l_1 - x_4)\frac{l_2}{h_6}\frac{(h_5 - h_4)}{p}\right]. \qquad (10)$$

Inspection of Eq. (10) shows that, for a load placed between L_6 and L_9, M_d varies directly as l_2 or the distance of the load from L_6; this variation may therefore be represented by a straight line $E'F'$ in Fig. 35, the ordinate $J'F'$ being given by Eq. (10). If P equals 1 pound, that equation becomes, for the example treated:

$$M_d = \frac{90}{180}(180 + 82.5) - \frac{90}{75} \cdot 45 - (82.5 + 180 - 120)\frac{90}{75}\left(\frac{10}{30}\right)$$
$$= 20\tfrac{1}{4} \text{ foot-pounds.}$$

The stress corresponding to this moment is compressive. The influence line is completed by connecting the points F' and G' by a straight line.

The final influence line (Fig. 35) shows that the greatest tension in C_4L_4 occurs when the distance $C'E'$ is covered by the uniform load; the greatest compression is found when spans l_2 and l_3, and the portion $A'C'$ of span l_1 are covered.

It is to be remembered that the shaded areas shown in Fig. 35 are moment areas, and must be divided by a lever arm to furnish stresses.

The influence line for any web member, such as U_2L_2, not lying below the cable members U_3U_4, etc., is also found by the use of Eq. (9), precisely as in the case of C_4L_4; but the terms in that equation which involve H and V disappear.

Art. 13. Influence Line for Web Members of the Cantilever Arm L_6L_9.

The influence line for the members C_8L_8 of the cantilever arm is shown in Fig. 37. The necessary parts of the

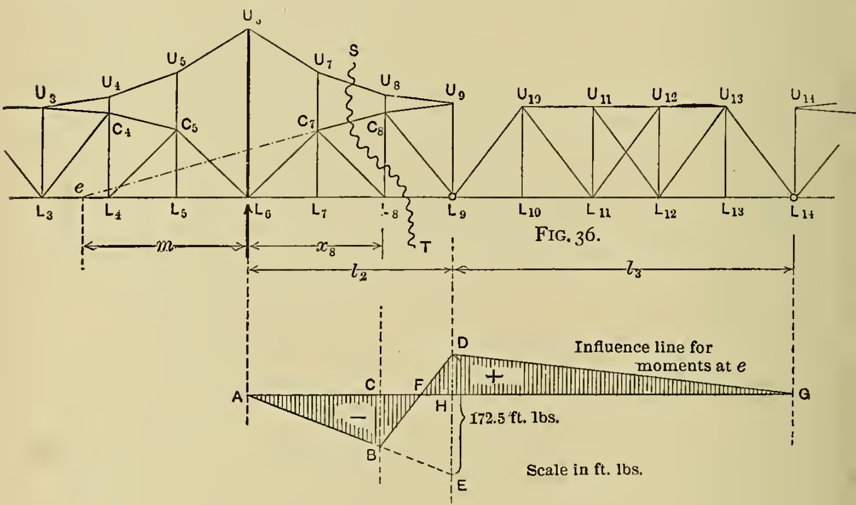


FIG. 36.

FIG. 37.

structure for determining that line are shown in Fig. 36. Loads placed on the anchor span l_1 cause no stresses in this member.

The stress in C_8L_8 is found by passing the section ST shown, and dividing the moment of the proper external

forces taken about e as a center, by the lever arm of C_8L_8. As in previous cases, therefore, the moment influence line for the moment M_e of the external forces taken about e will be drawn, since the stress in C_8L_8 may be found directly from it. The horizontal and vertical components of stress in U_7U_8 are denoted by H and V respectively, while m and x_8 are the distances shown in the figure. If a load P, of one pound, be placed at L_8, the equation of moments about e, treating U_7U_8 as the external force, will be

$$M_e = H \cdot h_8 + V(m + x_8). \quad . \quad . \quad . \quad . \quad (11)$$

But

$$H = \frac{P(x_8)}{h_6} \qquad \text{and} \qquad V = H \tan \alpha,$$

if α represents the angle between U_7U_8 and the horizontal. Therefore,

$$M_e = \frac{P \cdot x_8 \cdot h_8}{h_6} + \frac{P \cdot x_8(m + x_8)}{h_6} \tan \alpha. \quad . \quad , \quad (12)$$

Eq. (12) shows that M_e varies directly as x_8; since this term appears in the first degree, the equation may be represented by the straight line BA in Fig. 37, the ordinate BC having the value furnished by Eq. (12). For the structure under consideration, Eq. (12) becomes

$$M_e = \frac{60 \times 45}{75} + \frac{60(82.5 + 60)}{75} \frac{1}{3} = 74 \text{ foot-pounds.}$$

The influence line passes through A, a point on the base line, for a load at that point causes no stress in C_8L_8.

If a load P of 1 pound be placed at L_9, the equation for M_e becomes

$$M_e = \frac{P \cdot l_2 \cdot h_8}{h_6} + \frac{P \cdot l_2 \cdot (m + x_8)}{h_6} \tan \alpha - P(l_2 + m) . \quad (13)$$

Inspection of Eq. (13) shows that the first two terms of the right-hand member represent a continuation of the line expressed by Eq. (12). The third term shows that there must be subtracted from each ordinate of that line between L_8 and L_9, the value of P multiplied by its distance from e.

In Fig. 37 there must be subtracted from the ordinate EH the moment $P(90+82.5)=172.5$ foot-pounds.

The final influence line is therefore $ABDG$; the area FDG indicates a tensile stress, and the area ABF a compressive stress.

Art. 14.—Suspension Cantilever with Two Points of Support at Intermediate Pier.

Fig. 38 shows the suspension cantilever, treated in the preceding articles, but in which the post U_6L_6 has been

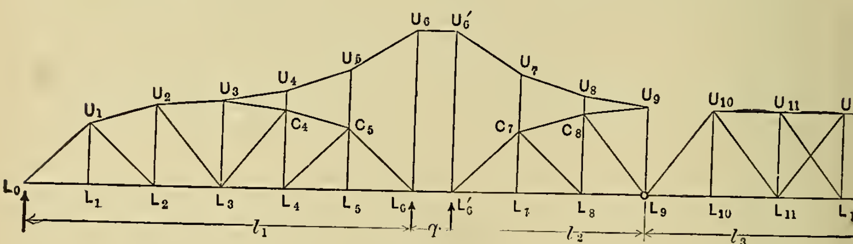


FIG. 38.

replaced by two columns, connected at top and bottom by horizontal members U_6U_6' and L_6L_6', but otherwise with no

bracing between them. No shear can then be transferred past L_6' to the left or past L_6 to the right. No change occurs, thefore, in any of the preceding constructions in determining R_0 or any of the stresses in the structure; it is necessary to observe that the open panel L_6L_6' is to be treated as a point, or entirely neglected. Changes occur, however, in determining the reactions R_6 and R_6' at L_6 and L_6' respectively; the stresses in the members U_6U_6' and L_6L_6' must also be found. These conditions are precisely the same as those for the ordinary cantilever structure,* supported on towers.

---

* See Page 158.

CHAPTER V.

DEFORMATION OF TRUSSES.

THE members of a framed structure when stressed suffer strains or deformations and these strains cause displacements at all points in the structure. These displacements may be found by graphic methods if the strains in the members are known. The deformation of any member may be expressed by the following equation:

$$\Delta l = \frac{Sl}{AE} + \varepsilon tl, \qquad \qquad (1)$$

in which Δl is the change in length of any member, expressed in inches, being positive for tensile stress and negative for compressive stress, S the total stress in the member, expressed in pounds, A the area of cross-section of the member in square inches, l the length of the member in inches, E the coefficient of elasticity in pounds per square inch, ε the thermal linear coefficient of expansion of the material in the member, per degree Fahrenheit, and t the change in temperature in degrees Fahrenheit. In order to find the displacement of the points of a structure it is necessary to consider the simultaneous strains or stresses caused by one position of loading and not the maximum strains or stresses, since the latter occur for different positions of the loading.

Art. 1.—The Displacement or Williot Diagram.

A simple statically determinate truss is a structure formed in general by adding to some triangle as a base element two additional members for each panel point in the frame. The problem of determining the displacement of any point in a simple statically determinate framework after the latter has been subjected to stress may therefore be treated in the most general way by obtaining the displacement of a panel point such as b (Fig. 1), which

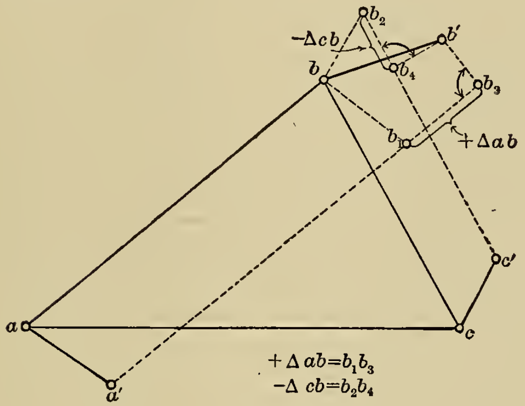


FIG. 1.

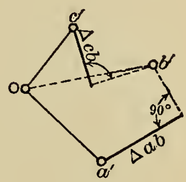


FIG. 2.

is connected to the member ac by the bars ab and bc. The member ac is also a member of the complete framework, in which the points a and c are assumed to have already received the known displacements aa' and cc' shown in the figure. By the application of stress the members ab and cb are assumed to receive deformations of $+\Delta ab$ and $-\Delta cb$, the positive sign representing an elongation.

Considering then the triangle abc as still unstressed, the points a and c of the bar ac will first suffer displacement to the positions a' and c'. This motion displaces

the point b of the bar ab to b_1, bb_1 being equal and parallel to aa'; and it also displaces the point b of the bar bc to b_2, bb_2 being equal and parallel to cc'. The member ab is increased in length, however, by an amount $+\Delta ab$, and this increase is represented by drawing b_1b_3 equal to $+\Delta ab$ and in the direction of a' to b_1. cb, however, is shortened by an amount $-\Delta cb$ and the point b_4 is found by laying off on $c'b_2$, in the direction of b_2 to c', the distance $b_2b_4 = \Delta cb$. As the point b cannot occupy the two positions b_3 and b_4 simultaneously, its final position must be at the intersection of the members $a'b_3$ and $c'b_4$, and this intersection may be obtained by rotating those members about a' and c' as centres respectively. It will, however, be sufficiently accurate for the small deformations generally found to replace these arcs of circles by straight lines drawn at right angles to the radii at their extremities. In this case, these lines are b_3b' and b_4b', intersecting at b'. The displacement of the original point b is then fully shown, both in magnitude and direction, by the heavy line bb'.

The preceding operation is more conveniently applied in a diagram entirely separate from the truss figure, and the displacement of the point b is again shown in Fig. 2, which is known as a displacement or Williot diagram. In that figure each displacement is referred to some fixed point known as a pole and represented by the letter O. This pole represents also the zero displacement of a point of the framework, taken originally as immovable or fixed. The displacement of any other panel point is measured by its displacement from the pole O.

In Fig. 2, therefore, Oc' and Oa' are drawn through O respectively parallel and equal to cc' and aa' to represent the displacements of the points c and a. At c', Δcb is then drawn parallel to bc and in the proper direction to represent the shortening of the member bc; and similarly, Δab is drawn parallel to ab, and in the direction a to b, to

represent the lengthening of that member. At the ends of these lines perpendiculars are erected and their intersection at b' shows the displacement Ob' of the point b. In Fig. 2 the line Ob' must be the same in direction and magnitude as bb' of Fig. 1.

This method is easily applied to the case of a truss-crane, shown in Fig. 3, for which it is required to find the displacement of the peak e, the point of support a being fixed in position. It is also assumed that the member ab suffers no change in direction. The crane is supporting only a load W at the peak.

In displacement work the following notation will in general be employed. On the truss diagram the panel points will always be designated by small letters, a, b, c, etc., while the members themselves will always be represented by numerals 1, 2, 3, etc. The deformations $\Delta 1$, $\Delta 2$, $\Delta 3$, will, however, each be marked on the displacement diagram itself only by the number of the member for which it represents the deformation. For instance, the member ab in Fig. 3 is known as member 1 in that figure, and its deformation in Fig. 5 is also represented by 1. The displaced position of any point will be represented by the primed letter of the point, such as a', b', c'. The displacement of such points from the pole O will represent their actual displacement.

Before obtaining the displacements of the points of the structure, it is necessary to determine the strains in the members. For this purpose the stress diagram shown in Fig. 4 must first be drawn. Table I shows in detail the length of the members. The intensities of stress existing in those members are obtained by dividing the total stresses scaled from Fig. 4 by the respective areas of cross-section, and the last column in the table shows the positive and negative strains. The coefficient of elasticity has been assumed at 30,000,000 pounds per square inch. It is the usual practice in dis-

TABLE I.

Member.	Length in Inches.	Intensity of Stress in Lbs. per Sq. In.	Deformation in Inches.
$ab=1$	120	-5000	$-.02$
$ac=2$	144	$+7000$	$+.034$
$bc=3$	144	-5000	$-.024$
$cd=4$	120	$+7000$	$+.028$
$bd=5$	180	-5000	$-.03$
$de=6$	96	$+7000$	$+.022$
$be=7$	220	-5000	$-.037$

$E=30,000,000$ pounds per square inch

placement work to employ the gross sections of tension members in place of the net sections, that is, to make no allowance for rivet-holes, and then to reduce slightly the

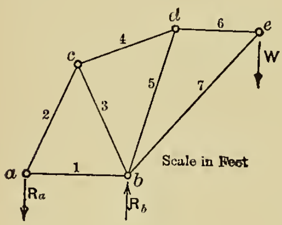


FIG. 3.

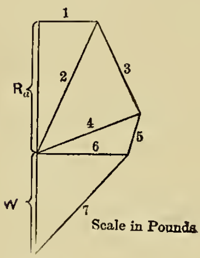


FIG. 4.

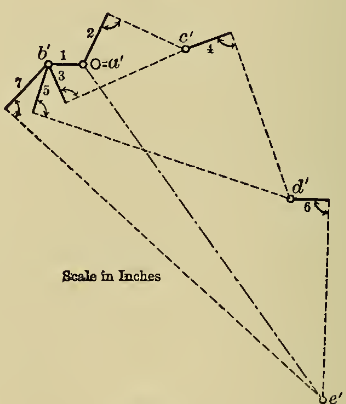


FIG. 5.

value of the coefficient of elasticity to a value of perhaps 28,000,000 or 29,000,000 pounds per square inch.

It has already been assumed in this problem that the member *ab* suffers no rotation in position but remains hori-

zontal and that the point a remains fixed in position. The point $O = a'$ is therefore chosen as the pole in Fig. 5. The displacement of b with relation to that pole is found by drawing 1, Fig. 5, parallel to 1, Fig. 3, and equal to $-.02$ inch. The member 1 is in compression, and therefore the displacement of b with regard to a is in the direction of b to a, as shown in Fig. 5. If ab were in tension, b' would be displaced to the right of the pole O. The displacement of the point c may now be found by drawing lines parallel to 2 and 3, equal to the deformations in those members and through the points a' and b'. The intersection of the perpendiculars erected at the ends of 2 and 3 will then determine the point c'.

The displacement of the point d is next obtained by drawing through the points b' and c' lines parallel to 4 and 5, representing in magnitude and direction the deformations in those members. The intersection of the perpendiculars to those lines erected at their extremities furnishes d'. The line connecting O with d' then represents the displacement of d with respect to O. In precisely the same way the displacement of the point e is found by drawing lines parallel to 6 and 7 through d' and b' and erecting perpendiculars at their extremities. The displacement of e with regard to a is therefore represented in Fig. 5 by the line Oe'. The displacement of any other point with regard to a is found by connecting the pole with any other point. In the problem, Oe' measures .24 inches.

The problem just solved is more simple than the usual cases of displacement found in bridge-trusses, for in those structures no member such as ab remains fixed in position. In simple bridge-trusses one end of the structure is usually fastened to a pin connection about which the entire truss may rotate, and the other pin end usually rests on rollers which permit that end panel point to move in one fixed

line only In the case of bridge-trusses, therefore, it is usual in the first place to consider any member as fixed in direction and one end of it fixed in position, then to determine the displacements of all the points in the structure with regard to that member alone. The structure, deformed and considered rigid in that condition, is then rotated about the end pin in such manner as to fulfil the condition that the other end moves in the proper line. Two separate constructions are therefore necessary for the case of bridge-trusses. The first construction is exactly similar to that already explained. The second construction involves the representation of the displacements of a rigid figure when rotated about some fixed point.

Art. 2.—Rotation of a Rigid Figure about a Point.

If a rigid figure such as the shaded triangle abc of Fig. 6 is rotated slightly, its motion may be expressed as one of rotation about an instantaneous centre I. If the angle of rotation for the triangle abc is α, the figure then takes the position $a'b'c'$. The lines aa', bb', cc' represent the displacements in position of the points of the triangle; they are respectively perpendicular to the lines Ia, Ib, and Ic. Fig. 7 represents a displacement diagram of a kind similar to that previously explained, O being chosen as a pole from which all displacements are to be measured. Let there be drawn as rays emanating from this pole the distances $a''O$, $b''O$, and $c''O$ respectively equal and parallel to aa', bb', and cc', these distances representing the changes in position of the corners of the rigid frame. Connecting the points a'', b'', and c'' by the lines $a''b''$, $b''c''$, and $c''a''$ it will then be found that the figure $a''b''c''$ is similar to the original frame abc, but turned exactly at right angles to it, for $a''O$ is perpendicular to aI; $b''O$ to bI, and $c''O$ to cI; and $a''O:aI::b''O:bI::c''O:cI$.

It will be evident in examining Fig. 7 that if the points a'' and c'' had been known, the position of the point b'' could have been determined by drawing upon the line $a''c''$ a figure similar to the rigid figure abc. In the general problems of trusses, if the rotation of any two points is known, the rotation of all others may be found. It will be found in statically determinate trusses that the rotation of two points is always known. In general, one point

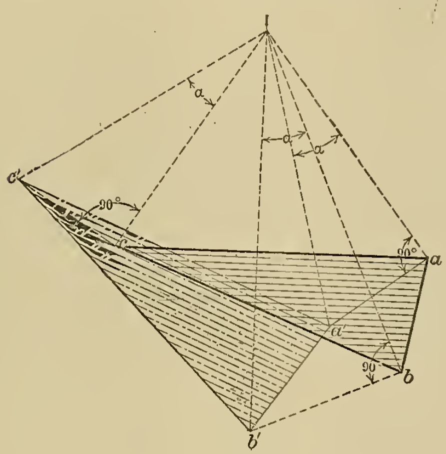


FIG. 6.

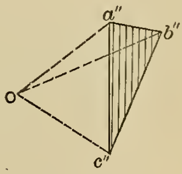


FIG. 7.

such as a will be the centre of rotation, while the amount of the displacement of some other corner such as c, when subjected to the conditions of the problem, will furnish the rotation of the second point.

If a point suffers two displacements, both of which are measured from the same pole O, the final displacement of that point, with regard to O, may be found by a triangle of displacements precisely similar to that of the triangle of forces. In Fig. 8 let $a''O$ represent the displacement of the point a of any figure found by the rotation of the structure about some point, and let Oa' represent the

displacement of the same point when the figure is deformed by stress. The final displacement of a is then represented by $a''a'$ measured in the direction as read, and found by closing the triangle of displacements $a''Oa'$. It is to be

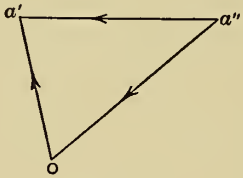


FIG. 8.

noted that the direction of the resultant in the triangle is opposite to that of the two component displacements.

In future notation the displacement of any point caused by stress will always be represented by the primed letter, such as a'. The displacement caused by rotation will always be represented by the second prime, such as a''. The final displacement must then always be read in the direction of a'' to a'.

Art. 3.—Deformations of a Bridge-truss.

The preceding analysis may now be applied to a bridge-truss in order to find the displacements of its panel points when it is subject to loading. A steel railroad-truss having eight panels of 30 feet each with a depth of truss at the centre of 40 feet, as shown in Fig. 9, will be taken. The other truss dimensions are shown in the figure. The deformations due to both dead and live load will be determined. The loading will be considered uniform, that being sufficiently accurate for the purpose. The moving train will be taken as covering the entire span. The dead loads or own weight are taken at 400 pounds per linear foot of span for the rails and other pieces that constitute

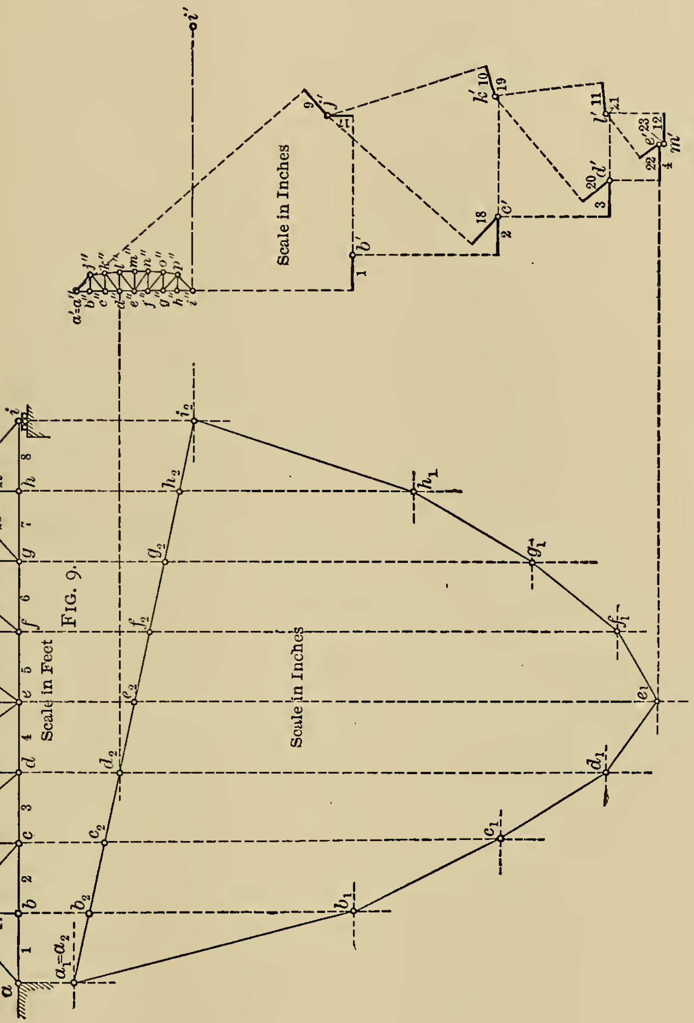


FIG. 9.

FIG. 10.

FIG. 11.

Truss Dimensions.

8 panels at 30 feet = 240 feet c. to c. end pins.
Height of bj = 27 feet; ck = 36 feet; dl = 40 feet.

the track; at 400 pounds per linear foot for the steel floor beams and stringers, and 1600 pounds per linear foot for the weight of trusses and bracing. The moving train load will be taken at 4000 pounds per linear foot. This will make the panel loads for each truss as follows:

Lower-chord dead load,	$30 \times 800 = 24,000$	lbs. per panel
Lower-chord moving load,	$30 \times 2000 = 60,000$	" " "
Total load on lower chord	$= 84,000$	" " "
Upper-chord dead load,	$30 \times 400 = 12,000$	" " "

The structure is a "through" bridge, hence all moving loads rest on the lower chord. The stresses in the trussed members due to the combined uniform dead and moving load are easily found by the graphical method, one diagram only being needed. It is not necessary to show this diagram.

Table I shows, then, the total stresses, the intensities of stresses, the lengths l, and the deformations Δl in the members. The displacement diagram (Fig. 10), may then be drawn in a manner similar to that for the trussed crane, assuming me as fixed in direction and one point of it, m, fixed in position. It is usually preferable to take as the member fixed in position a bar which it is known in advance suffers little displacement. This avoids an unwieldy diagram and explains the choice in the present instance. The point m' (Fig. 10) represents, therefore, the pole, and the displacement of e with respect to m is shown as $m'e'$. The displacement of l' may then be found by drawing lines parallel to 12 and 22 through the points m' and e' respectively, equal to the deformations in those members and erecting perpendiculars at their ends. The intersection of these perpendiculars gives the displacement of l at l'. The displacement of d may then be found by drawing lines parallel to 4 and 21 through e' and l' and erecting

TABLE I.*

Member.	Total Stress in Member in Pounds.	Intensity of Stress in Lbs. per Sq. Inch.	Length in Inches.	Δl Deformation in Inches.	Member.	Total Stress in Member in Pounds.	Intensity of Stress in Lbs. per Sq. Inch.	Length in Inches.	Δl Deformation in Inches.
1	+373,300	+ 12,000	360	+ .154	8	+ 373,300	+9600	360	+ .123
2	"	"	"	"	7	"	"	"	"
3	+480,000	"	"	"	6	+ 480,000	"	"	"
4	+540,000	"	"	"	5	+ 540,000	"	"	"
9	−502,300	− 9,000	472	− .152	16	−502,300	−7200	472	− .122
10	−501,000	− 9,500	376	− .128	15	−501,000	−7600	376	− .102
11	−544,800	− 10,000	363	− .129	14	−544,800	−8000	363	− .093
12	−576,000	"	360	− .129	13	−576,000	"	360	"
17	+ 84,000	+ 9,000	324	+ .104	29	+ 84,000	+7200	324	+ .083
18	+143,500	+ 10,000	472	+ .169	28	+ 143,500	+8000	472	+ .135
19	− 12,000	− 1,000	432	− .015	27	− 12,000	− 800	432	− .012
20	+ 93,720	+ 7,400	562	+ .148	26	+ 93,720	+5920	562	+ .118
21	+ 12,000	+ 1,000	480	+ .017	25	+ 12,000	+ 800	480	+ .014
22	+ 60,000	+ 4,800	600	+ .102	24	+ 60,000	+3840	600	+ .082
23	− 12,000	− 1,000	480	− .017		− 12,000			

$E = 28,000,000$ pounds per square inch.

* It will be noted that the right half of this table is similar to the left half, but that the intensities of stress for that part of the table are but 80 per cent. of those shown for the members indicated in the left half. This was done in order to avoid symmetry of the displacement diagram and would not, in general occur in practice.

perpendiculars at their ends. The displacement of k is next found by drawing through l' and d' lines parallel to 11 and 20 respectively, equal in magnitude and in the proper direction to represent their deformations and erecting perpendiculars at their ends. Panel points c, j, b, and a are then considered in that order.

a' shows the final displacement position of a, the left-hand end of the truss. The displacements of the panel points of the right half of the truss are not shown in detail, in order to avoid confusion of the diagram, but they would be found in a precisely similar manner. The diagram shows only the displacement of the right end i at i'.

The figure has so far been drawn upon the assumption that the point m is fixed in position and the direction of

me fixed. Actually, however, it is the point a which remains fixed in position and the point e is displaced to the amount $a'e'$. The diagram also shows the displacement of i to be $i'e'$, whereas the point can move only in a horizontal line. The distorted figure, now considered rigid, must therefore be so rotated about the point a that the point i falls in the line in which the end rollers move. By the methods of the previous article, a' at once becomes a'', and it suffices merely to find the displacement of one other point in order to locate all the other displacements. The point i moves in a line at right angles to ai when the truss rotates about a. In Fig. 10, therefore, the line $a''i''$ is drawn at right angles to ai, and its intersection with $i'i''$, drawn horizontal, determines the final displacement of i. $i'i''$ is drawn horizontal, for that is the only possible direction of its displacement. The points a'' and i'' now furnish two points for the diagram from which the displacement of all other points in the figure, as it is rotated about a, may be found. It is simply necessary to draw upon $a''i''$ as a lower chord a reproduction of the main truss, $a''j''k''l'' \ldots i''$, every line of which must be at right angles to the corresponding line of Fig. 9.

The final displacement of every point is now found as the resultant of the two previous displacements, these latter always being measured in the direction of the second primed letter toward the first primed letter. For instance, the displacement of the point e is downward from e'' to e'.

In general it is sufficient to determine merely the vertical projections of the displacements or the deflections of the panel points carrying the moving load, for these deflections indicate the extent to which the truss must be cambered. They may be found by projecting the positions of the primed letters e', d', c', etc., to the left until they intersect at e_1, d_1, c_1, etc., verticals dropped from the panel points. The open polygon obtained by connecting

these points may be closed by a line a_1i_2 in the manner of closing a funicular polygon. The deflection of any panel point is then represented by the intercept in the polygon directly below that point, e_2e_1, for instance, being the deflection of the point e.

Art. 4.—Displacement Diagram for a Three-hinged Arch.

Fig. 12 represents a three-hinged arch for which it is desired to obtain the displacements of all the panel points when the structure carries load. The stresses, the sectional areas, the lengths, and the deformations of the members are not given in tabular form, but the strain to which each member is subjected is shown to scale in the truss diagram by a heavy line on each member, the minus sign near each member representing a compressive strain, while the plus sign indicates an elongation.

Treating first the left half of the arch, it is assumed that kd suffers no change in direction and that the point d remains fixed in position. Under that assumption the usual displacement diagram, Fig. 13, for the left half of the arch may be drawn. A line parallel to 1 is drawn vertically downward from d' (the pole) to represent the compressive strain in kd. At d' a line parallel to 2 is then drawn to indicate the displacement of j. The displacement of c is then found by drawing lines through j' and d' parallel to 4 and 3 respectively and finding the intersection of the perpendiculars erected at the extremities of these lines. The displacements of the panel points i, b, h, and a are then found by treating the strains in the order in which the members are numbered.

The right half of the arch is then to be treated in precisely the same manner, but it is sufficient to indicate in Fig. 13 simply the displacement of the point g, namely g'. It is now necessary to determine the rotation of the two halves

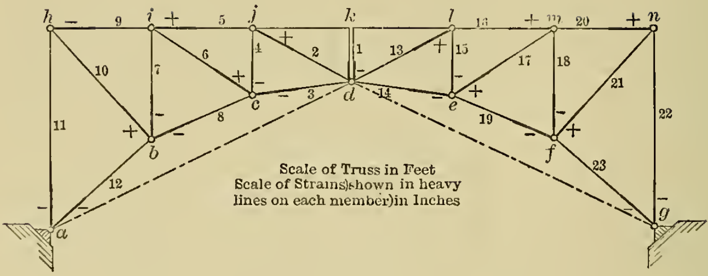

Scale of Truss in Feet
Scale of Strains shown in heavy lines on each member in Inches

FIG. 12.

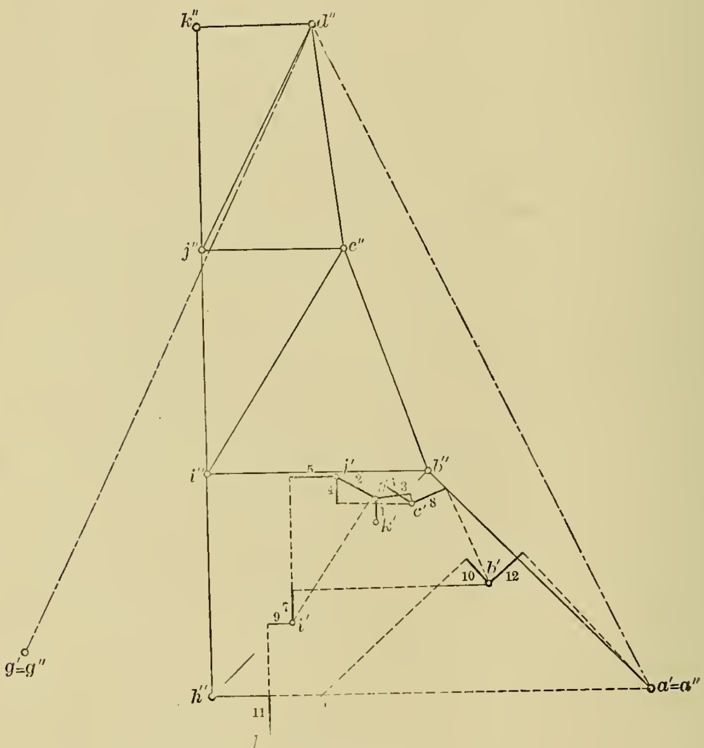


FIG. 13.

of the arch about the points a and g as centres respectively in such manner that the point d may coincide for both halves.

Since the two parts of the structure rotate respectively about the fixed points a and g, the points a' and g' of Fig. 13 become at once a'' and g''. Treating the left half of the arch first, it is seen that d rotates at right angles to the line ad, connecting a with d. In the displacement diagram (Fig. 13) the path of this rotation is indicated by $a''d''$, a line drawn through a'' at right angles to ad. Similarly the right half of the arch must rotate about g, and the path of the rotation of d about g is represented by a line drawn through g'' at right angles to gd of Fig. 12. The intersection of these lines fixes at once the final displacement of d at d''. Since two final points of displacements have been found for each half of the arch, the displacement of all other points may be found by drawing a figure between d'' and a'' precisely similar to the right-hand half of the arch, but with every line at right angles to it. It is indicated in Fig. 13 by $d''k''h'\,a''$. The displacement of any point in that part of the structure is obtained by a line connecting the double-primed with the single-primed letters, the displacement of h for instance being $h''h'$.

In order to avoid confusion the displacement of the points on the right half of the figure are not shown. There would be required simply a figure drawn between g'' and d'' similar to the right half of the arch, but with every line at right angles to it.

Displacements due to changes of temperature are treated in precisely the same manner as the displacements due to stress deformations. The changes in length caused by the stresses are merely replaced by the changes in length caused by the change in temperature.

Art. 5.—Displacement Diagram for Truss with Sub-divided Panels.

In Fig. 14 is shown a Williot or displacement diagram, for dead load only, for a truss with sub-divided panels, having a span length of 350 feet; the complete design of this truss is given in Chapter VI.

TABLE I.

Member.	Total Fixed Load Stress in Pounds.	Area of Cross-section. Square Inches.	Length in Inches.	Distortion of Member in Inches.
L_0L_1	+ 234,000	46	350	+0.0635
L_1L_2	+ 234,000	51	350	+ .0572
L_2L_3	+ 340,000	71	350	+ .0525
L_3L_4	+ 382,000	81	350	+ .059
L_4L_5	+ 382,000	83	350	+ .0575
L_5L_6	+ 424,000	90	350	+ .0589
L_0U_1	− 371,000	97.23	556	− .0758
U_1U_2	− 356,000	83.25	366	− .056
U_2U_3	− 400,000	90.25	366	− .058
U_3U_4	− 453,000	99.1	354	− .0578
U_4U_5	− 453,000	99.1	354	− .0578
U_5U_6	− 448,000	99.1	350	− .056
U_1L_1	+ 33,000	17.9	432	+ .0284
U_1L_2	+ 168,000	34.0	556	+ .0982
U_2L_2	− 98,000	39.4	540	− .0163
U_2L_3	+ 78,000	20.1	643	+ .0893
U_3L_3	− 32,000	19.4	648	− .0382
U_3M_4	+ 90,000	25.4	477	+ .0603
U_4M_4	− 20,000	12.9	363	− .0201
M_4L_4	+ 33,000	24.5	324	+ .0156
M_4L_5	+ 57,000	20.1	477	+ .0483
M_4U_5	+ 30,000	12	556	+ .0496
U_5L_5	− 7,000	19.4	756	− .00976
U_5M_6	+ 36,000	19.3	515	+ .0343
U_6M_6	− 20,000	12.9	378	− .0209
M_6L_6	+ 33,000	14.4	378	+ .031

Coefficient of Elasticity = 28,000,000 lbs. per sq. inch.

In Table I are shown for each member the total dead load stress, the area of cross-section in square inches, and

the length in inches. In the last column there have been calculated the changes in length of the various members due to dead load, the coefficient of elasticity being assumed at 28,000,000 lbs. per square inch. These changes in length are written along the members of the truss in Fig. 16.

The Williott diagram is begun by assuming panel point o as fixed in position at o' and the displacement of p with regard to o is taken in the direction of op, so that the new position of p becomes p'. The displacements of all other panel points is then found with reference to that displacement. Since the member op suffers an elongation of .031″, the line $o'p'$ must be drawn downwards to represent that displacement.

The displacement of the panel point l at l' may next be found by drawing a line from p' parallel to pl with a length of .0589 inches.

The displacement of m at m' is next found by drawing a line $l'm'$ .00976″ long parallel to the lower chord member lm and erecting a perpendicular at the extremity of that line. There is then drawn at o' a line parallel to om, having a length of .0343″, and a perpendicular erected at its extremity. The intersection of the two perpendiculars thus drawn, determines the position of the point m'.

In precisely the same manner the displacement of other panel points may be found; j might be determined next, and then h, k, g, etc.; after the left half of the truss is completed, the same methods of construction are employed for the right half of the truss.

The final points to be located will be the two ends of the truss, a and a_1 at a' and a_1' respectively, a_1' being found 0.68 inches to the right and 0.99 inches above a'.

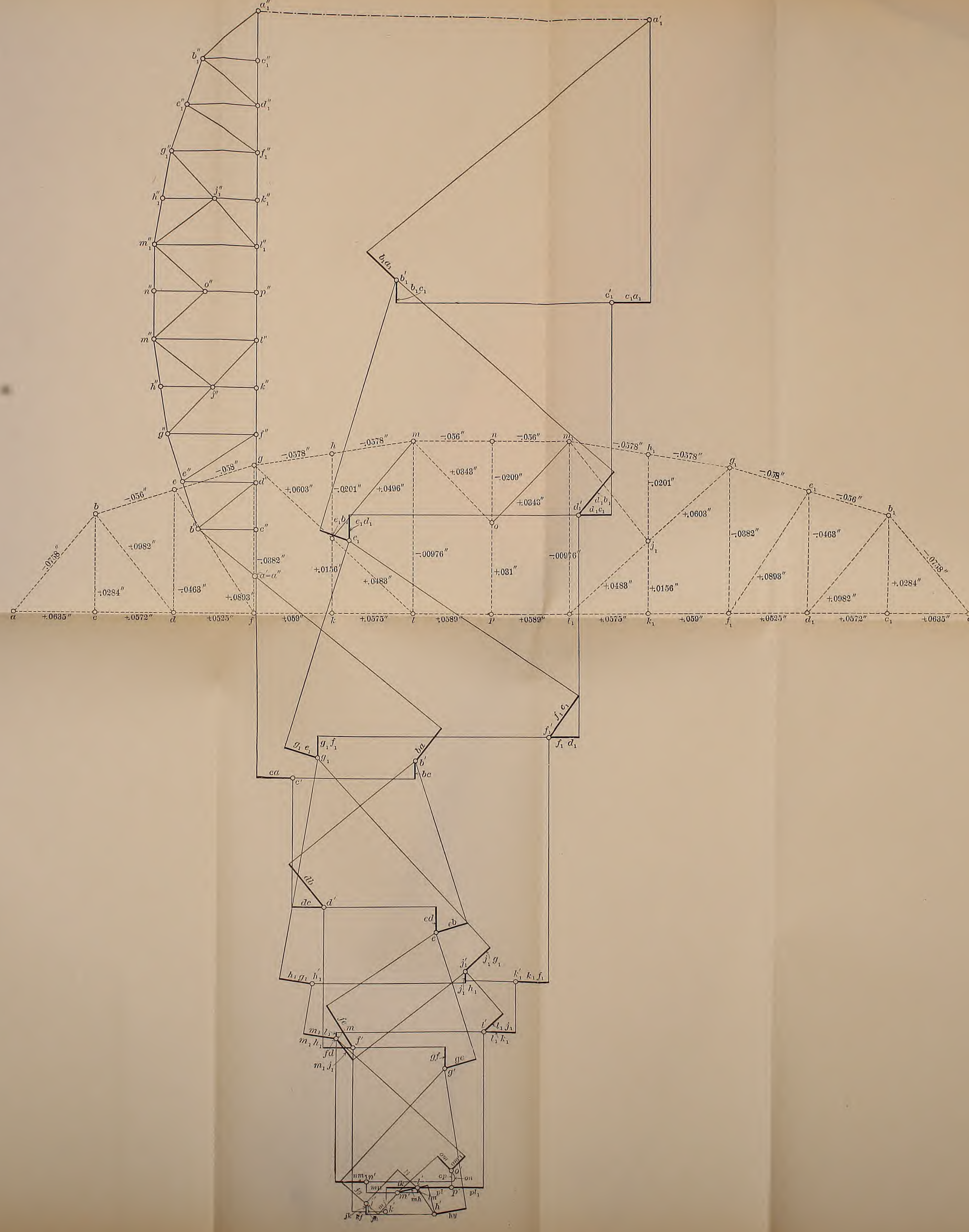
Since the point a is fixed in position, its true displacement must be zero and the point marked a' must be a'' or the origin about which to rotate the figure of the truss in order to

determine all other displacements; since the point a_1 rests on rollers which can move only in a horizontal line, the true displacement of a_1 is found by drawing a horizontal line through a_1' until it intersects a vertical erected at a''; that intersection is marked a_1'', and the distance from a_1'' to a_1' measures the true and final displacement of the point a_1; viz., 0.68 inches to the right

As in all previous cases the line $a''a_1''$ is the base line upon which there is now to be drawn a reproduction of the figure of the truss, the line $a''a_1''$ being used as the lower chord line; all panel points of this truss are marked with the second primed letters.

The final and true displacement of any point of the truss under the dead load may now be found, since it is that displacement which is measured from the point marked by the second primed letter to the point marked by the single primed letter; for instance, the point p is actually displaced by the distance p'' p', or a vertical displacement downwards of 1.57 inches and a horizontal displacement toward the right of 0.34 inches.

As in the previous examples of the Williott diagram, the strains or distortions of the members of the truss are shown in the diagram as heavily shaded lines, and they are marked with the letters indicating to which member of the truss they pertain; the perpendiculars erected at the extremities of the heavier lines are shown with a light line.



−.0738″
+.0982″
+.0284″
−.0468″
+.0893″
−.0382″
+.0603″
−.0201″
+.0496″
+.0848″
−.0209″
+.0348″
+.0156″
+.0488″
−.00976″
+.031″
+.0635″
+.0572″
+.0525″
+.059″
+.0575″
+.0589″
−.056″
−.058″
−.0578″

CHAPTER VI.

THE DETAILED DESIGN OF A RAILROAD BRIDGE.

THE design of the bridge will be based on the following data;* in engineering practice these would be furnished by the railroad company for which the bridge is to be built:

Length of span between centres of end pins. Panel length, 29′ 2″, and number of panels, 12	350′	0″
Distance between centres of trusses	18′	0″
Distance between centres of stringers	7′	6″
The centre height U_5L_5 is	63′	0″
The intermediate height U_3L_3 is	54′	0″
The end height U_1L_1	36′	0″

The bridge is a through one, for single track, and the form of truss is of the type known as the broken upper chord with subdivided panels, as is clearly shown on the stress sheet, Plate I. As specified by the railroad company, all of the material of the bridge is to be of medium steel; all the rivets are to be $\frac{7}{8}$ inch, except that $\frac{3}{4}$-inch rivets may be used in the flanges of 12-inch channel posts, in horizontal struts at the middle of posts, and in the overhead lateral or sway bracing. The structure is to be designed according to Cooper's specifications of 1896.

---

* The authors are indebted to the American Bridge Company of New York for the stress sheet and detail drawings of this bridge; the original design was in charge of Mr. O. E. Hovey, Engineer of Design.

Art. 1.—Stresses in the Structure.

The stresses in the structure are caused by its own or dead weight and by the live or moving load. The dead load may be divided into two parts, the weight of the floor system consisting of rails, ties, track-fastenings, stringers, and floor-beams, and the trusses and bracing. Excerpts from Cooper's specifications relating to these loadings are the following:

§ 23. *All the structures shall be proportioned to carry the following loads:*

1. The weight of metal in the structure.

2. A floor weighing 400 pounds per linear foot of track to consist of rails, ties, and guard timbers only.

These two items, taken together, shall constitute the dead load.

3. A "live load" on each track, supposed to be moving in either direction, consisting of 2 "consolidation" engines, coupled and followed by a train load, distributed as shown on diagram E 40 (or 100,000 pounds equally distributed on two pairs of driving-wheels, spaced seven and a half feet, centre to centre).†*

The maximum stresses due to all positions of either of the above "live loads" of the required class, and of the "dead load," shall be taken to proportion all parts of the structure.

§ 27. *Variation in temperature, to the extent of 150° F., shall be provided for.*

§ 28. *When the structures are on curves, the additional effect due to the centrifugal force of trains shall be considered as live load. It will be assumed to act 5 feet above base of rail,*

---

* For a diagram of locomotive E 40, see page 100.

† This part of the specification is usually employed in determining the maximum bending moments and shears in the floor system. In the example under consideration this alternative loading has been neglected.

and will be computed for a speed of 50 – 2d miles per hour; d being the degree of curve.

§ 29. *All parts shall be so designed that the stresses coming upon them can be accurately calculated.*

§ 28 does not apply to the structure under consideration, and the requirements of § 27 may be fulfilled by placing one end of the bridge upon a nest of movable rollers.

Art. 2.—Determination of Dead-load Stresses.

The weight per linear foot of the trusses, not including the floor system but including all the lateral and cross bracing, may be approximately estimated by means of the following formula: $W = al$, where W is the weight per linear foot of both trusses and bracing in pounds, a is a numerical coefficient, and l is the length of span in feet. In the bridge under consideration a may be taken as 7.75. The weight of the two trusses then becomes 2700 pounds per linear foot, or 39,380 pounds per panel per truss. This weight must be equally divided between the upper and lower panel points. The specifications require the weight of track to be taken at 400 pounds per linear foot, while the remainder of the floor system may be assumed to weigh 500 pounds per linear foot; hence these loads become 5830 and 7290 pounds per panel per truss respectively. They act at the lower panel points.

The upper panel-point loading is then 19,690 pounds, but the lower panel-point loading becomes 32,810 pounds.

The dead-load stresses are easily determined by graphic methods as shown in Fig. 1; for that figure only those web members shown in full lines are assumed to act, the dotted members acting as counters.* It is unnecessary to explain the details of the figure; but to insure the accuracy

---

*See Chap. II, Art. 18, for the treatment of subdivided panels.

of the graphic diagram it is important to check analytically the last stress found. The stress in L_5L_6, for instance, is so found by passing a section through U_5U_6, U_5M_6, and L_5L_6 and taking the centre of moments at U_5.

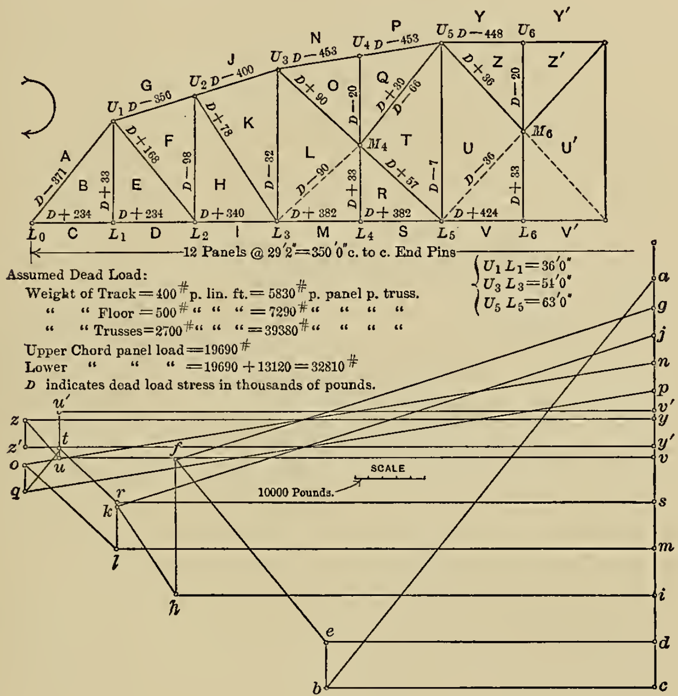


FIG. 1.

The compression stresses in L_3M_4 and M_4U_5 are found by assuming U_3M_4 not to exist and are quickly determined by the method of sections.

Art. 3.—Determination of Live-load Stresses in the Chords.

The position of the moving load causing the greatest bending moment at any point in a truss or beam is found by the following criterion,* it being supposed for convenience that the train passes along the truss from right to left with W_1 as the front load:

$$\frac{l'}{l}=\frac{W_1+W_2+W_3\ldots W_n'}{W_1+W_2+W_3\ldots W_n}. \quad . \quad . \quad . \quad . \quad (1)$$

In eq. (1), l' represents the distance from the left end of the truss to the centre of moments, i.e., to the point in question, and l the total length of the truss; $W_1+W_2\ldots W_n'$ the sum of the loads on l' and $W_1+W_2\ldots W_n$ the sum of the loads on l.

By the method of sections the stress in a chord member is found by dividing the maximum bending moment at the proper panel point by the normal distance (or lever-arm) from the panel point to the chord member in question. This method is directly applicable to the truss considered, although the modification furnished in Chap. II, Art. 20 is necessary in the case of the chord members in the subdivided panels.

By drawing a reaction influence line both the truss reaction and the bending moment at any point for any position of the moving load may be at once measured; the diagram should be made on a large scale in order to attain the highest practicable degree of accuracy.

* See eq. (6), Chap. II, Art. 6.

TABLE I.

Member.	Wheel-load Number.	At Panel Point.	Stress in Pounds.	Remarks.
L_0L_1	5	L_1	$+333,000$	Train advancing from right to left
L_1L_2	5	L_1	$+333,000$	do.
L_2L_3	13	L_2	$+480,000$	do.
L_3L_4	17	L_3	$+537,000$	do.
L_4L_5	17	L_3	$+537,000$	do.
L_5L_6	23	L_5	$+590,000$	do.
L_0U_1	5	L_1	$-528,000$	do.
U_1U_2	13	L_2	$-502,000$	do.
U_2U_3	17	L_3	$-562,000$	do.
U_3U_4	20	L_4	$-631,000$	Criterion of the subdivided panel
U_4U_5	20	L_4	$-631,000$	
U_5U_6	26	L_6	$-624,000$	

By applying the criterion, eq. (1), and completing these diagrams, the results shown in the preceding table will be obtained. The table exhibits both the positions of the concentrations for the greatest bending moments and the greatest stresses in the various chord members.

The uniform load is treated as a uniform series of loads of 25,000 pounds each; i.e., load No. 23 in the table indicates the fifth load of this series.

Art. 4.—Determination of Live-load Stresses in Web Members.

The methods of Chap. II, Art. 16 were used to determine the position of the loading causing the maximum stresses in the web members of this truss, the modifications necessary for subdivided panels being given in Art. 19 of the same chapter.

The application of those methods gives all the results, both as to positions of loading and the greatest web stresses, exhibited in Table II:

TABLE II.

Member.	Wheel-load Number.	At Panel Point.	Stress in Pounds.	Remarks.
U_1L_2	4	L_2	+257,000	Train advancing from right to left
U_2L_2	3	L_3	−161,000	do.
U_2L_3	3	L_3	+160,000	do.
U_3L_3	3	L_4	−103,000	do.
U_3M_4	3	L_4, for panel L_3L_4	+204,000	do.
M_4L_5	5	L_5, for panel L_3L_5	+168,000	do.
U_5M_4	When L_4M_4 is max.		+ 62,000	do.
	3	L_3, for panel L_4L_5	+104,000	Train advancing from left to right
M_4L_3	5	L_3, for panel L_3L_5	+ 82,000	do.
U_5L_5	3	L_6	− 79,000	Train advancing from right to left
U_5M_6	3	L_6, for panel L_5L_6	+173,000	do.
M_6L_5	3	L_5, for panel L_5L_5	+136,000	do.

The members U_1L_1, M_4L_4, and M_6L_6 are simply hangers. They carry in tension the maximum floor-beam load, which occurs when wheel No. 5 is directly at the hanger.

The members U_4M_4 and U_6M_6 are simply supporting struts; they carry in compression the dead load assumed to exist at their upper ends, but they carry no live load.

The horizontal struts M_3M_4 and M_5M_6 are intended simply to prevent flexure of the long column into which they frame. They are subject to no direct stresses of either tension or compression.

The determination of the maximum tension in the vertical members U_3L and U_5L_5 has already been treated in full in Art. 21, Chap. II.

Art. 5.—Determination of Wind Stresses.

Cooper's specifications prescribe:

"§ 24. *To provide for wind stresses and vibrations . . . the bottom lateral bracing in through bridges, for all spans up to 300 feet, shall be proportioned to resist a lateral force of 450 pounds for each foot of the span; 300 pounds of this to be treated as a moving load, and as acting on a train of cars, at a line 8.5 feet above base of rail.*

" . . . *the top lateral bracing in through bridges for all spans up to 300 feet shall be proportioned to resist a lateral force of 150 pounds for each foot of span.*

"*For spans exceeding 300 feet add in each of the above cases 10 pounds for each additional 30 feet of span.*"

In accordance with these specifications, the assumed wind load for the bottom chord will be 470 pounds per linear foot, and for the upper chord 170 pounds per linear foot.

Upper Lateral Bracing.

The stresses for the upper chord lateral bracing are found in the most expeditious way by analytical methods.*

The bracing may be considered a truss of 12 panels lying in a horizontal plane with its abutments at the feet of the end posts. The loading is fixed in position and equal to $29.1 \times 170 = 4950$ pounds per panel, the reaction at the end post being 29,700 pounds. The secant of the angle of inclination between the web members and the perpendicular to the plane of the main truss is 1.9. The stresses in the web members are then found by multiplying

* For the treatment of wind bracing, as exemplified in this chapter, the reader is referred to the authors' "Metallic Bridges."

the shear in the respective panels by this secant, and they are given on the stress sheet. It should be noted that in this treatment the panel of the portal is considered a part of the wind bracing, although the true lateral bracing in that panel is replaced by a portal frame. This involves no error in determining the stresses of the webbing of the system, and is on the side of safety for the chord members. The stresses in the chord members are easily found by the method of sections.

Lower Lateral Bracing.

The lower lateral bracing is designed for a moving load of 300 pounds per lineal foot and a fixed load of 170 pounds per lineal foot. The stresses in the chord members of that system are easily found by multiplying the stresses in the chord members of the upper lateral system by the ratio between the loads of the two systems, i.e., $\frac{470}{170}=2.76$. The sum of the fixed and moving loads is taken, since the truss must be entirely covered by the moving load for maximum chord stresses. The resulting stresses are shown on the stress sheet.

The stresses in the webbing are found by taking the stationary load the same as for the upper chord bracing and combining its effects with those due to the moving load. The following examples illustrate the method employed:

Web Member in Panel L_0L_1.

The stress in this member has its maximum value when the entire truss is covered by the moving load. The reaction then becomes 52,500 pounds, and the shear in the panel is $52,500-(29.1\times 300)/2=48,150$ pounds. The stress is then $48,150\times 1.9=91,500$ pounds. To this must be added

the stress due to the stationary wind load, viz., 5100 pounds. The sum of these two quantities may then be assumed to be carried equally between the two intersecting web members within the panel, causing compression in the one and tension in the other member.

The resultant stress for each is therefore $\frac{1}{2}(91,500 + 51,000) = 71,250$ pounds.

Web Member in Panel L_1L_2.

For this member, $^{10}/_{11}$ of the bridge, or 318.1 feet, should be covered by the uniform moving load.* The shear in the panel is then $43,400 - 3620 = 39,780$ pounds, and the stress is 75,500 pounds. The stress due to the stationary load is 42,000 pounds; hence the final stress in the member is $\frac{1}{2}(75,500 + 42,000) = 58,750$ pounds.

The other web members of this lateral system are treated in precisely the same manner.

Art. 6.—Design of Lower Chord Members.

The specifications which apply to the design of the lower chords are the following:

§ *30. All parts of the structures shall be proportioned in tension by the following allowed unit stresses:*

Bottom chords, main diagonals, counters, and long verticals (forged eye-bars)—for live loads 10,000 pounds per square inch and for dead loads 20,000 pounds per square inch.

§ *36. The stresses in the truss members or trestle posts from the assumed wind forces need not be considered except as follows:*

1st. When the wind stresses on any member exceed one quarter of the maximum stresses due to the dead and live loads

* The student may check this by the method of influence lines.

upon the same member. The section shall then be increased until the total stress per square inch will not exceed by more than one quarter the maximum fixed for dead and live loads only.

Member L_5L_6.

The stresses in this member, as taken from the stress sheet, are:

Dead-load stress	$= +424{,}000$	pounds.
Live-load stress	$= +590{,}000$	"
Wind stress	$= \pm 402{,}000$	"

Neglecting the wind stress, the area of cross-section required would be 80.2 square inches, as shown:

$$424{,}000 \div 20{,}000 = 21.2 \text{ square inches.}$$
$$590{,}000 \div 10{,}000 = \underline{59.0} \quad " \quad "$$
$$80.2 \quad " \quad "$$

The actual intensity of stress in the member would then be $(424{,}000 + 590{,}000) \div 80.2 = 12{,}600$ pounds per square inch. The unit stress due to wind would then be

$$\frac{402{,}000}{80.2} = 5010 \text{ pounds,}$$

which exceeds by $5010 - 3150 = 1860$ pounds, one quarter of the unit stress due to the dead and live load. The sectional area of the chord member, therefore, must be increased until the intensity of stress does not exceed $12{,}600 \times \frac{5}{4} = 15{,}750$ pounds. The combined dead, live, and wind loads is 1,416,000 pounds. Hence the required area of cross-section will be

$$\frac{1{,}416{,}000}{15{,}750} = 89.9 \text{ square inches.}$$

The member will then be composed of six eye-bars, each 8 inches deep by $1\frac{7}{8}$ inches thick, the area of whose combined cross-section will be 90 square inches.

Other lower chord members are treated in exactly the same way, and it will suffice to consider one more only.

Member L_3L_4.

Dead-load stress	$= +382,000$ lbs. @ 20,000 lbs.	$= 19.1$	sq. in.
Live-load stress	$= +537,000$ lbs. @ 10,000 lbs.	$= 53.7$	"
		72.8	"
Wind stress	$= +\ \ 356,000$ lbs.		
Total stress	$= +1,275,000$ "		

Neglecting wind stress, the average stress in the member would be

$$\frac{382,000 + 537,000}{72.8} = 12,620 \text{ pounds per square inch.}$$

Adding 25 per cent. to this stress on account of wind, the final section required becomes

$$\frac{1,275,000}{(12,620 + 3,154)} = 80.8 \text{ square inches.}$$

The member will be composed of six eye-bars, each $8 \times 1\frac{11}{16}$ inches, whose combined area will be 81 square inches.

The lower chord members in the first two panels of the truss are subjected to a more sudden loading than those in the other panels; it is therefore customary to form these members either of shapes suitable to resist compression, such as channels, or stiffen them by latticing the eye-bars together. In the present case the latter plan will be adopted; the two centre eye-bars, four being necessary, will be tied together by $2\frac{1}{2} \times \frac{1}{2}$-inch lattice bars.

Art. 7.—Design of Upper Chord Members.

The following specifications apply to the design of the upper chord:

§ 33. Compression members shall be proportioned by the following allowed unit stresses:

Chord segments:

$$P = 10{,}000 - 45\frac{l}{r} \text{ for live-load stresses;}$$

$$P = 20{,}000 - 90\frac{l}{r} \text{ for dead-load stresses.}$$

End posts are not to be considered chord segments. All posts of through bridges:

$$P = \ \ 8500 - 45\frac{l}{r} \text{ for live-load stresses;}$$

$$P = 17{,}000 - 90\frac{l}{r} \text{ for dead-load stresses,}$$

in which expressions P is the allowed stress in pounds per square inch of cross-section, l is the length of compression member in inches; r is the least radius of gyration of the section in inches.

No compression member, however, shall have a length exceeding 125 times its least radius of gyration.

§ 70. The unsupported width of plates subjected to compression shall not exceed 30 times their thickness, except cover-plates of top chords and end posts, which will be limited to 40 times their thickness.

§ 90. In compression chord sections the material must mostly be concentrated at the sides, in the angles and vertical ribs. Not more than one plate, and this not exceeding ½ inch

in thickness, shall be used as a cover-plate, except when necessary to resist bending stresses or to comply with § 70.

Member U_5U_6.

Dead-load stress $= -448,000$ pounds.
Live-load stress $= -624,000$ "

It will now simplify the design work to reduce the dead-load stress to the equivalent live-load stress; for this member the live-load stress, plus one half the dead-load stress, becomes 848,000 pounds.

The allowed unit stress is not a constant quantity for compression members, but it depends upon the radius of gyration of the cross-section of the member. It becomes necessary, therefore, in designing, to assume some trial value for the radius of gyration. In the rectangular or box form of section adopted for upper chord members, it will usually be found that the radius of gyration has a value of about 0.4 the depth of the side plates, the depth of the member being taken slightly greater than would be the case in a vertical column on account of the bending induced by its own weight. This depth is usually about one-twelfth to one-fifteenth the length.

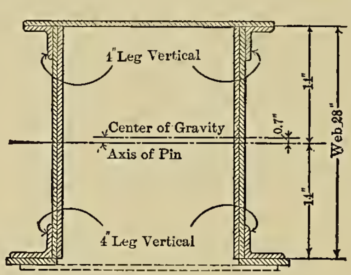
In the case of U_5U_6, the depth of the member may be taken as 28 inches; the assumed radius of gyration becomes 11.2, and since l is 350 inches, the assumed unit stress becomes

$$P = 10,000 - 45\frac{350}{11.2} = 8594 \text{ pounds per square inch.}$$

The approximate area required is therefore

$$848,000 \div 8594 = 98.7 \text{ square inches.}$$

A trial cross-section must now be formed from angles and plates of about this area, and tested to ascertain whether the assumed radius of gyration must be modified. The upper angles in the trial section should be made as light as possible, so that the cover-plate may be largely balanced by the heavy bottom angles. Flat bars are frequently added to the horizontal legs of the lower angles for the same purpose. In this manner the centre of gravity of the section may be brought down sufficiently near to mid depth to give the space needed inside the chord for the eye-bar heads, the pin axis being made to pass through the centre of gravity of the section. At the same time there is gained the incidental but important advantage of an increased moment of inertia. If the chord were subject to no bending from its own weight, the axis of every pin should pass through the centre of gravity of the section. It may be shown * that this flexure cannot be satisfactorily met and neutralized by the direct stress, particularly if



CHORD-SECTION U_5U_6

FIG. 2.

the chord is considered as continuous. It is best therefore to reduce the bending stresses by making the chord depth as great as possible and placing the axis of the pin either through the centre of gravity of the section or but slightly below it.

* See Art. 69, Burr's "Course on Stresses in Bridge and Roof Trusses."

Let the following section (shown in Fig. 2) be tested:

		Sq. In.
1 cover-plate	$30'' \times \frac{5}{8}''$	$= 18.75$
2 angles	$4'' \times 3\frac{1}{2}'' \times \frac{9}{16}'' =$	7.80
2 web plates	$28'' \times \frac{1}{2}''$	$= 28.00$
2 " "	$28'' \times \frac{7}{16}''$	$= 24.50$
2 angles	$5'' \times 4'' \times \frac{9}{16}''$	$= 9.50$
2 bars	$6'' \times \frac{7}{8}''$	$= 10.50$
		99.05

The position of the centre of gravity of this section must first be determined by taking moments about the upper edge. The following calculation will determine its position:

Moment of cover-plate	$18.75 \times 0.32 = 5.9$
" " upper angles	$7.80 \times 1.91 = 14.9$
" " web plates	$52.50 \times 14.63 = 768.0$
" " lower angles	$9.50 \times 27.51 = 261.8$
" " bars	$10.50 \times 29.07 = 306.0$
	Total = 1,356.6

Dividing this sum by the total area of the section determines the distance of the centre of gravity from the upper edge, i.e.,

$$\frac{1356.6}{99.05} = 13.8 \text{ inches.}$$

The moment of inertia of the section must now be found about this centre of gravity as an axis:

Cover-plate,	$\frac{30 \times (.63)^3}{12} + [18.75 \times (13.5)^2] = 3,420.6$
Upper angles,	$2 \times 5.71 + [7.80 \times (11.9)^2] = 1,114.4$
Web plates,	$\frac{1.875 \times (28)^3}{12} + [52.50 \times (.8)^2] = 3,463.6$
Lower angles,	$2 \times 6.27 + [9.50 \times (13.7)^2] = 1,794.5$
Bars,	$\frac{12 \times (.875)^3}{12} + [10.50 \times (15.3)^2] = 2,460.7$
	Total = 12,253.8

The radius of gyration is equal to the square root of the quotient obtained by dividing the moment of inertia by the area, that is,

$$r = \sqrt{\frac{I}{A}},$$

where r = the radius of gyration of section,
I = moment of inertia of section,
and A = area of section.
In the present case

$$r = \sqrt{\frac{12,253.8}{99.05}} = 11.1 \text{ inches.}$$

This is so nearly the value assumed that it will not be necessary to repeat the calculations with the true value of r. If there had been a considerable difference between the assumed and the actual values of r it would have been necessary to repeat the calculations with such changes in the trial section as would furnish satisfactorily correct results.

It must be determined whether the moment of inertia may not be smaller about the vertical axis of symmetry of the section; if this be the case, this smaller value of r must be used to determine the allowed intensity of stress. The calculations for this moment of inertia are as follows:

Cover-plate,	$\frac{.625 \times (30)^3}{12} =$	1,405
Upper angles,	$2[4.07 + (12.53)^2 \times 3.9] =$	1,236
Web plates,	$2\left[\frac{(\frac{15}{16})^3 \times 28}{12} + 26.25 \times (11.03)^2\right] =$	6,404
Lower angles,	$2[11.26 + (13.12)^2 \times 4.75] =$	1,656
Bars,	$2\left[\frac{.875 \times (6)^3}{12} + 5.25 \times (13.5)^2\right] =$	1,944
	Total =	12,645

This moment of inertia is larger than the one found for the horizontal axis, and it need not therefore be further considered.

The distance from back to back of the lower and upper angles will be made $28\frac{1}{4}$ inches, being slightly greater than the depth of side plates in order to prevent their interference with the cover-plate or lower bars during manufacture. The axis of the pins will be placed $14\frac{1}{2}$ inches from the upper edge of the cover-plate, thus allowing a small counter moment from the direct stress due to a lever-arm of 0.7 inch. The other upper chord members are treated in precisely the same manner. The stress sheet, Plate 1, shows in sufficient detail the forms of cross-section adopted.

End Post L_0U_1.

The end post is designed under a different provision of the specifications, the allowed intensity of stress for live load being found by the following formula:

$$p = 8500 - 45\frac{l}{r}.$$

The length of the end post between pins is 558 inches, but it may be assumed that the effective length is only one half of this, since the post is supported at its centre in one direction by an inclined strut, and in the direction at right angles by the portal bracing. Assuming $l = 279$ inches and $r = 11.1$ inches, the allowed intensity of stress becomes 7370 pounds per square inch.

Dead-load stress	$= -371,000$	pounds
Live-load stress	$= -528,000$	"
($\frac{1}{2}$ dead + live) load stress	$= -713,500$	"

Therefore $\frac{713,500}{7,370} = 96.9$ square inches required.

The stress sheet indicates the section adopted for this member. It will be found on examination that the requirements of the specifications concerning the unsupported widths of plates in compression have been fulfilled.

Art. 8.—Design of Floor-beam Hangers.

U_1L_1, L_4M_4, L_6M_6.

These members are built from shapes rather than from bars with forged ends, in order that the floor-beams may be attached to them with the same details that are used at other panel points. The following specifications apply:

§ *30. Floor-beam hangers and other similar members (in tension) liable to sudden loading (plates and shapes), net section at 6000 pounds per square inch.*

Long verticals in tension (plates or shapes), net section at 9000 pounds per square inch for live loads and 18,000 pounds per square inch for dead loads.

U_1L_1, L_4M_4, and L_6M_6 are long verticals, and they are covered by the second of the preceding provisions.

Member U_1L_1.

Dead-load stress	= + 33,000	pounds
Live-load stress	= + 84,000	"
($\frac{1}{2}$ dead + live) load stress	= + 100,500	"

$\frac{100500}{9000} = 11.17$ square inches required.

The following 15-inch channels spaced 14 inches back to back more than satisfy this requirement:

Two 15-inch channels weighing 32.95 pounds per lin. ft. @ 9.69 square inches = 19.38 square inches gross or 17.88 square inches net, held together by single latticing $2\frac{1}{4} \times \frac{3}{8}$ inches.

This is much more metal than is needed, but the member is 36 feet long and requires increased stiffness on that

account. As these same channels must be used for a number of the main web members of the truss, they will also be used for these long verticals.

Members L_4M_4 and L_6M_6.

Both of these members are subjected to the same stresses, and since they occupy similar positions in the truss, they will be designed exactly alike.

Dead-load stress $= +$ 33,000 pounds
Live-load stress $= +$ 84,000 "
($\frac{1}{2}$ dead $+$ live) load stress $= +$ 100,500 "
$\frac{100500}{9000} = 11.16$ square inches required.

A narrower channel than 12 inches cannot be used on account of the floor-beam connection; it will be necessary to use two 12-inch channels, each weighing 24.51 pounds per linear foot, with a total gross area of 14.42 square inches and a net area of 12.86 square inches tied together by single latticing $2\frac{1}{4} \times \frac{3}{8}$ inches. These members will be continued vertically upward to form the members M_4U_4 and M_6U_6 respectively; their sections are ample to carry the compressive stresses imposed.

Art. 9.—Design of Vertical Main Web Members or Posts.

U_2L_2, U_3L_3, U_5L_5.

These main vertical posts are all in compression and therefore subject to the following specification:

§ 33. *All posts of through bridges shall be proportioned by the following allowed unit stresses:*

$$P = 8{,}500 - 45\frac{l}{r} \text{ for live-load stresses;}$$

$$P = 17{,}000 - 90\frac{l}{r} \text{ for dead-load stresses.}$$

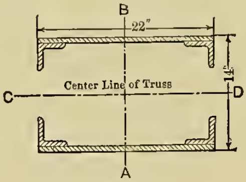
Member U_2L_2.

Dead-load stress	$= -\ \ 98{,}000$ pounds
Live-load stress	$= -161{,}000$ "
($\frac{1}{2}$ dead + live) load stress	$= -210{,}000$ "

By assuming the width of post at 22 inches, and the radius of gyration .35 of this width, the length of the post being 540 inches, the allowed intensity of stress becomes

$P = 8500 - \dfrac{45 \times 540}{7.7} = 5340$ pounds per square inch, and the

area of section required $\dfrac{210{,}000}{5{,}340} = 39.3$ square inches.



POST U_2L_2

FIG. 3.

The following trial section (Fig. 3) may be assumed:

2 plates $22'' \times \frac{9}{16}''$	$= 24.75$ square inches
4 angles $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{9}{16}''$	$= 14.60$ " "
	39.35 " "

The plates are spaced 14 inches back to back. The moment of inertia about the AB axis will be:

$$\text{Plates} = 2\left[\frac{\frac{9}{16} \times (22)^3}{12}\right] = 1000$$

$$\text{Angles} = 4[4.0 + (3.65 \times 10^2)] = 1476$$

$$\text{Total} = 2476$$

$$\therefore\ r = \sqrt{\frac{2476}{39.35}} = 7.95 \text{ inches.}$$

It will not be necessary to make use of the radius of gyration about the *CD* axis (in this case 6.48 inches), since the length of the column to be inserted in the formula, when considering this radius, is much reduced by the cross-bracing at its upper end. The allowed intensity of stress for this shortened column, taken in connection with the smaller radius of gyration, should always be tested.

In the present case the value of 7.95 inches for the *AB* radius checks the value assumed sufficiently near so that recalculation will not be necessary; the trial section will be adopted as it stands, using single $2\frac{1}{2}\times\frac{3}{8}$ inch lattice bars.

Member U_3L_3.

Dead-load stress $= -32{,}000$ pounds
*Live-load stress $= +25{,}000$ "
Live-load stress $= -73{,}000$ "

In the case of this member, the following specification relative to alternate stress becomes operative:

§ 35. *Members subject to alternate stresses of tension and compression shall be proportioned to resist each kind of stress. Both of the stresses shall, however, be considered as increased by an amount equal to $\frac{8}{10}$ of the least of the two stresses for determining the sectional areas by the usual allowed unit stresses.*

The equivalent live-load stress for U_3L_3 then becomes ($\frac{1}{2}$ dead $+ \frac{8}{10}$ tension live $+$ compression live) load stress $= -109{,}000$ pounds.

Assuming a width of 15 inches, and the radius of gyration .35 the width, the length of the column being 27 feet (not 54 feet, since it is supported in two directions at the centre) furnishes a unit stress of $8500-(45\times 324)/5.25=$

*See Chap. II, Art. 21, for the determination of this counter-stress.

5720 pounds per square inch. The required area will therefore be

$$\frac{109,000}{5,720} = 19.0 \text{ square inches.}$$

A trial section of 2–15 inch channels, weighing 32.95 pounds per linear foot, with a total area of section of 19.38 square inches, will be found to furnish a radius of gyration of 5.67 inches, and to fulfil all the required conditions. The spacing back to back of channels will be made uniform with that of the other hangers and posts at 14 inches, and the latticing will be single, $2\frac{1}{2} \times \frac{3}{8}$ inches.

Member U_5L_5.

Dead-load stress	$= - 7,000$	pounds
Live-load stress	$= +27,000$	"
Live-load stress	$= -79,000$	"

The equivalent live-load stress is $(\frac{1}{2} \times 7000 + \frac{8}{10} \times 27,000 + 79,000) = 104,100$ pounds compression.

The effective length of column may be assumed as one half of 63 feet, and since the stress in U_5L_5 is nearly the same as that in U_3L_3, the section belonging to the latter may be used for a trial; therefore $r = 5.67$ inches. The allowed intensity of stress then becomes: $p = 8500 - (45 \times 378)/5.67 = 5500$ pounds per square inch, and the area of cross-section required $104,100/5500 = 18.95$ square inches.

The trial section of 19.38 square inches may therefore be adopted as final, using as in the other cases a spacing of 14 inches back to back of channels and single latticing $2\frac{1}{2}$ inches by $\times \frac{3}{8}$ inch.

Art. 10.—Design of Main and Counter Web Members.

Tension Web Members, U_2L_3, U_3M_4, M_4L_5, U_5M_6.

The following specification applies to these tension web members:

§ *30. The allowed unit stresses in tension for main diagonals and counters shall be:*

10,000 pounds per square inch for live loads.
20,000 pounds per square inch for dead loads.

Member U_2L_3.

Dead-load stress = + 78,000 pounds
Live-load stress = + 160,000 "

The equivalent live-load stress is ($\frac{1}{2}$ dead + live) load stress = + 199,000 pounds.

The required area of cross-section will therefore be

$$\frac{199,000}{10,000} = 19.9 \text{ square inches}$$

and will be formed of 2 eye-bars 7 inches $\times$ $1\frac{7}{16}$ inches = 20.12 square inches.

The following table shows the method of design for the other web members.

Member.	Dead-load Stress in Pounds.	Live-load Stress in Pounds.	Equivalent Live-load Stress in Pounds.	Area Required, Square Inches.	Section Adopted.	Area of Final Section, Square Inches.
U_3M_4	+90,000	+204,000	+249,000	24.9	2 bars—$7'' \times 1\frac{13}{16}''$	25.38
M_4L_5	+57,000	+168,000	+196,500	19.65	2 bars—$7'' \times 1\frac{7}{16}''$	20.12
U_5M_6	+36,000	+173,000	+191,000	19.1	2 bars—$7'' \times 1\frac{3}{8}''$	19.26

Counter Web Members, L_2U_3, L_3M_4, L_5M_6, and M_4U_5.

The following specification applies to these members:

§ 45. *The areas of counters shall be determined by taking the difference in areas due to the live- and dead-load stresses considered separately; the counters in any one panel must have a combined sectional area of at least 3 square inches, or else must be capable of carrying all the counter live load in that panel.*

Member L_2U_3.

There is actually no counter stress in the panel L_2L_3, and the member L_2U_3 is only inserted to secure a margin of safety. In compliance with the specifications, therefore, it must have a cross-section of at least 3 square inches. One adjustable rod $1\frac{3}{4}$ inches square having an area of 3.06 square inches fulfils these requirements.

Member L_3M_4.

Dead-load stress $= -90,000$ pounds
Live-load stress $= +82,000$ "

The difference between the equivalent of the dead-load stress in terms of the live load and the live load will be $-\frac{1}{2}(90,000)+82,000=37,000$ pounds tension.

At 10,000 pounds per square inch a cross-sectional area of 3.7 square inches will be required. Two adjustable rods $1\frac{1}{2}$ inches square having an area of cross-section of 4.5 square inches will be used.

The stress in the remaining counter members are not completely determined statically, and the sectional areas of those members should be based on the undiminished live-load stresses in them.

Member L_5M_6.

Dead-load stress = − 36,000 pounds
Live-load stress = + 136,000 "

This member should be designed according to the last condition of the specification, viz., to be capable of carrying all the counter live load in the panel; the area required will therefore be 13.5 square inches, and will be composed of 2 adjustable bars $6 \times 1\frac{1}{8}$ inches having an area of 13.5 square inches.

Member M_4U_5.

Live-load stress = + 39,000 pounds
Dead-load stress = + 62,000 "

or

Dead-load stress = − 66,000 pounds
Live-load stress = + 104,000 "

This member will be designed according to the same condition as L_5M_6. In this case all the counter live-load stress equals 104,000 pounds, and at 10,000 pounds per square inch the area required would be 10.4 square inches, which is supplied by two 6×1 inch eye-bars having an area of 12 square inches.

This completes the design of the main members of the truss; the intermediate struts M_3M_4, M_5M_6, and the collision strut supporting the centre of the end post carry no definite stresses. Their form of section depends to a great extent upon their end connections, and they are usually built of light angles latticed together in box shape.

In the present case all these struts will be built of four $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inch angles, having a combined area of 10 square inches laced together in a box form by four sets of single lacings $2\frac{1}{4} \times \frac{5}{16}$ inches. The dimensions of the box forms will depend upon the connections which these struts make with other members.

Art. 11.—Combined Stresses.

§ *39. When any member is subjected to the action of both axial and bending stresses, as in the case of end posts of through bridges, or of chords carrying distributed floor loads, it must be proportioned so that the greatest fibre strength will not exceed the allowable limits of tension or compression on that member. If the fibre stress resulting from weight only, of any member exceeds 10 per cent. of the allowed unit stress on such member, such excess must be considered in proportioning the areas.*

In accordance with the above specification the chord members should be tested with reference to the bending stresses induced by their own weight. The condition of the ends or of the supported sections of these sections considered as beams is largely a matter of conjecture; they may be considered fixed at their ends or assumed entirely free to turn, or their condition may be assumed half way between. It is probably reasonable, at least, to assume these members fixed at their ends or supported sections, since the friction on the pins will tend to produce that condition.

It will be sufficient for present purposes to test one upper and one lower chord member for combined stresses. The method here adopted for finding the bending stress in the truss members, although always yielding safe results, is not exact. Strictly speaking, the bending effect of the axial stresses should be recognized.*

Member U_5U_6.

The cross-sectional area of this number is 99.09 square inches, but allowing 20 per cent. additional for rivets,

* For the exact theory, see Burr's "Resistance of Materials," edition 1903, p. 181.

latticing, and other details, the weight per inch of length will be 33.8 pounds. The length between centres of end pins is 350 inches; therefore the maximum bending moment M, assuming the member to have fixed ends, will be at the centre, and equal to 518,000 inch-pounds.

The moment of inertia I of the section is 12,254; therefore k, the intensity of stress in the extreme fibres, is, by the following formula,

$$k = \frac{Md}{I} = \frac{518,000 \times 14}{12,254} = 592 \text{ pounds per square inch.}$$

This is less than 10 per cent. of the allowed compressive intensity in the member, and it may therefore be neglected.

Member L_5L_6.

The dimensions of one eye-bar are 8 inches deep $\times 1\frac{7}{8}$ inches thick $\times$ 350 inches long; the weight per inch of length will be $4\frac{1}{4}$ pounds, and the maximum bending moment at the centre, assuming fixed ends, will be 32,500 inch-pounds. The moment of inertia is 80; therefore the stress in the extreme fibres at the centre will be

$$\frac{32,500 \times 4}{80} = 1625 \text{ pounds per square inch.}$$

The permissible axial unit stress (see page 186) was 15,750 pounds per square inch, and the bending stress is so slightly in excess of 10 per cent. that it may be safely neglected.

There still remains to be treated the bending in the end post caused by the overturning effect of the wind; the maximum bending will occur at the connection of the curved bracket of the portal to the end post and at the foot of the end post, since that member may be considered fixed at the lower end, and may be treated precisely as in

the case of a post supporting a roof-truss by the aid of brackets (Chap. I, Art. 18). A point of contraflexure midway between the foot of the end post and the curved bracket may therefore be assumed to exist.

If it be supposed that the wind loads of the upper chord are carried equally to both end posts, the maximum bending moment becomes $\frac{1}{2}$ the wind reaction by $\frac{1}{2}$ the distance along the post measured from its foot to the bracket connection; in this case 14,875 pounds $\times$ 140 inches $=$ 2,082,500 inch-pounds. The moment of inertia about the axis of bending is about 12,500, and the distance from the neutral axis to the extreme fibre of the upper angles is 15 inches, it being remembered that the bending takes place in the plane at right angles to the vertical plane through the axis of the truss. The stress in the extreme fibre will then be

$$\frac{2{,}082{,}500 \times 15}{12{,}500} = 2500 \text{ pounds per square inch.}$$

At the same time the stress in the web plates, which constitute the greater portion of the cross-section of the post, will be $(2{,}082{,}500 \times 11.5)/12{,}500 = 1920$ pounds per square inch, due to this same bending.

This stress is a wind stress, and is therefore subject to § 36 of the specifications. The average stress in the end post due to dead and live loads is $(371{,}000 + 528{,}000)/97.25 = 9250$ pounds, and one quarter of this is 2310 pounds; therefore the bending due to wind may be safely neglected. As a further margin of safety it should be noted that the point of greatest bending due to wind does not coincide with the point of greatest bending due to the simple compression in the member.

Art. 12.—Design of Stringers and Floor-beams.

The following specifications apply to these members:

§ *30. The allowed unit stress in tension, for the bottom flanges of riveted cross-girders (net section), shall be 10,000 pounds per square inch, and for the bottom flanges of riveted longitudinal plate girders used as track stringers (net section), 10,000 pounds per square inch.*

§ *40. In beams and plate girders the compression flanges shall be made of the same gross section as the tension flanges.*

§ *41. Riveted longitudinal girders shall have, preferably, a depth not less than one tenth the span.*

§ *42. Plate girders shall be proportioned upon the supposition that the bending or chord stresses are resisted entirely by the upper and lower flanges, and that the shearing or web stresses are resisted entirely by the web plate; no part of the web plate shall be estimated as flange area.*

The distance between centres of gravity of the flange area will be considered as the effective depth of all girders.

§ *43. The webs of plate girders must be stiffened at intervals of about the depth of the girders, wherever the shearing stress per square inch exceeds the stress allowed by the following formula:*

Allowed shearing stress =

$$\frac{12{,}000}{1+\dfrac{H^2}{3{,}000}},$$

where H = ratio of depth of web to its thickness; but no web plates shall be less than $\frac{3}{8}$ of an inch in thickness.

The following specifications for riveting also apply to the design of these members:

§ *37. The rivets in all members other than those of the*

floor and lateral systems must be so spaced that the shearing stress per square inch shall not exceed 9000 pounds, nor the pressure on the bearing surface (diameter × thickness of the piece) of the rivet-hole exceed 15,000 pounds per square inch.

The rivets in all members of the floor system, including all hanger connections, must be so spaced that the shearing stresses and bearing pressures shall not exceed 80 per cent. of the above limits.

In the case of field riveting (and for bolts) the above-allowed shearing stresses and pressures shall be reduced one third.

Rivets and bolts must not be used in direct tension.

Stringers.

The stringers are assumed 350 inches long and $32\frac{1}{4}$ inches back to back of angles, the depth of the stringer being taken a little less than one tenth of the span to permit the required track elevation. The actual depth of the web plate will be 32 inches, but the distance back to back of angles is increased by $\frac{1}{4}$ inch, so that the web will not project between the angles and cause difficulty in riveting the cover-plates. It should be noted that shallow stringers deflect more than deeper ones, thus tending to produce greater direct tensile stresses on the end-connection rivets.

The maximum bending moment due to live load will be found for that position of loading in which the centre of gravity of the load is situated as far on one side of the centre of the beam as the point of maximum bending is on the other side, and a wheel load will always be found over this point of maximum bending. In the present case this point is found 14.2 feet from the left end of the stringer under the second driver of the locomotive, as shown by Fig. 4.

The reaction at the left end of the stringer for this position of the loading is 43,800 pounds; the maximum bending moment is then found as follows:

$$M = 43{,}800 \times 14.2 - 10{,}000 \times 13 - 20{,}000 \times 5 = 392{,}000 \text{ ft.-lbs.}$$
$$= 4{,}704{,}000 \text{ in.-lbs.}$$

The dead-load bending moment at the same point may be taken equal to the maximum dead-load bending moment at the centre, and is found as follows:

The track is assumed to weigh 400 pounds per linear foot, and both stringers and bracing 375 pounds per linear

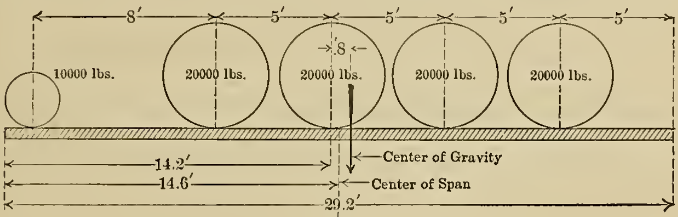


FIG. 4.

foot. The weight per linear inch per stringer will then be 32.29 pounds, and the maximum moment

$$\frac{32.29 \times (350)^2}{8} = 494{,}000 \text{ inch-pounds.}$$

The effective depth of the girder according to the specifications may be assumed at 30 inches, and since the allowed intensity of stress is 10,000 pounds per square inch, the required net section will be

$$\frac{4{,}704{,}000 + 494{,}000}{30 \times 10{,}000} = 17.3 \text{ square inches.}$$

This flange or chord area will be afforded by two angles $6 \times 6 \times \frac{9}{16}$ inches, having a gross section of 13 square inches $= 10.7$ square inches net, and one cover-plate $14 \times \frac{9}{16}$ inches,

having a gross section of 7.9 square inches = 6.8 square inches net. Total 17.5 square inches net.

Length of Cover-plate.

The length of the cover-plate will be determined by assuming that the bending moment follows a parabolic law. If L represents the total length of the stringer, l the length of the cover-plate, A the total section of the flange, and a the total area of all the cover-plates counting from the outside and including the area of the one under consideration, then

$$l = L\sqrt{\frac{a}{A}}.$$

Inserting the requisite quantities in the formula,

$$l = 29.2\sqrt{\frac{6.8}{17.3}} = 18.3 \text{ feet.}$$

At least 6 inches should be added to each end of the computed length of a cover-plate in which to place additional rivets, so that the plate may actually develop its full strength at the section where it is needed. The length of the cover-plate in this case will therefore be 19.3 feet.

Web Plate.

The maximum shear for both dead and live loads occurs at the end of the stringer. The live-load shear with the first driver at this point is 62,500 pounds. The dead-load shear is 5700 pounds, thus making the total shear 68,200 pounds. The working shear in the web plate may be taken at 5000 pounds per square inch of gross section. The required cross-section will then be 13.6 square inches,

and the web plate taken will be $32 \times \frac{7}{16}'' = 14.0$ square inches. According to the specifications, if the shearing stress exceeds $12{,}000/[1 + (H^2/3000)]$, intermediate stiffeners will be required. In the present case this formula has a value of $12{,}000/[1 + (73.2^2/3000)] = 4310$ pounds per square inch; whereas the actual intensity of shear in the stringer is 5000 pounds per square inch at the ends, with a constantly decreasing value towards the centre of the stringer; no intermediate stiffeners will therefore be required. The end stiffeners cannot be subjected to a rational analysis; in the present case they will be composed of two $6 \times 6 \times \frac{7}{16}$ inch angles, being chosen of such dimensions as to permit of making simple connections to the floor-beams. They are sometimes designed by requiring that their normal section must carry the end reaction with a certain specified unit stress (as 7000 pounds per square inch). In this case the normal section is 10.12 inches, and it is ample for the purpose.

Riveting.

The following rule will determine the pitch in the flanges of a girder, viz., the pitch of the riveting at any section of the flanges will be the quotient obtained by dividing the product of the vertical distance between the rows of riveting in the two flanges and the allowed stress in one rivet either for shear or bearing, by the maximum shear at the section. This rule is readily demonstrated by taking the difference between the bending moments at two adjacent rivets.

In the present case the pitch at the ends of the stringer will be

$$\frac{30 \times (5740 \times 80\%)}{68{,}200} = 2.1 \text{ inches.}$$

At 10 feet from the ends the dead-load shear will be 1925 pounds, and the maximum live-load shear, with the first driver over the section, will be 31,300 pounds.

The pitch at this point will therefore be

$$\frac{30 \times (5740 \times 80\%)}{31,300 + 1925} = 4.15 \text{ inches.}$$

The flange rivets have thus far been considered to carry the direct horizontal stress only, but those in the upper flange must also carry as a vertical load the weight of the driving-wheels as they pass. One driver weighing 20,000

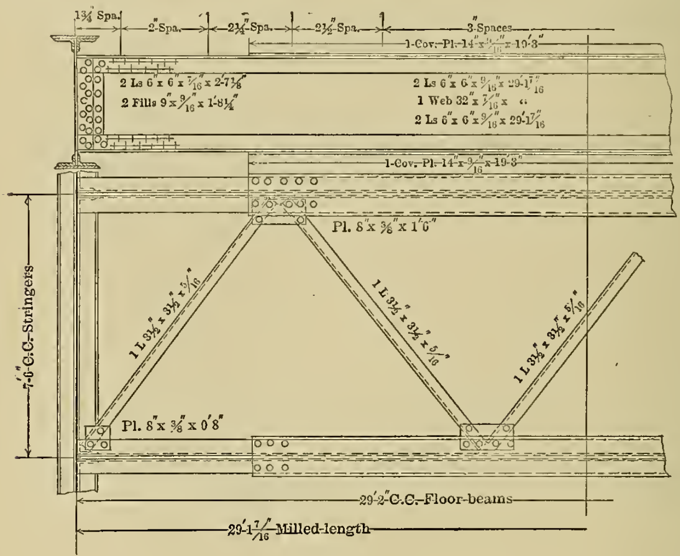


FIG. 5.

pounds may be supposed to be distributed over three ties, or over a distance of 3 feet, assuming 6-inch spaces between ties 8 inches wide. The greatest effect in increasing the pitch will be found at the centre of the girder. At this point 9 rivets would approximately be required in a distance of 3 feet, if the longitudinal stress alone existed: and 20,000/4592 = 4.4 rivets for the vertical load alone.

The proper number of rivets for this distance of 3 feet is then $\sqrt{9^2+4.4^2}=10$. It is seen that the effect of this vertical loading in increasing the number of rivets is very small. The spacing finally adopted is shown in Fig. 5; it varies from $1\frac{3}{4}$ inches at the ends to 3 inches at the centre, and more than provides for vertical load effect.

Rivets in Cover-plates.

The stress carried by the cover-plate will be 6.8 square inches $\times$ 10,000 pounds = 68,000 pounds. The permissible stress in the rivets will be determined for single shear, at 80 per cent. of 5410 pounds = 4328 pounds per rivet. Sixteen rivets will be required. They will be placed in four rows with 3 inches pitch for a distance of 15 inches from the ends of the plate, and 6 inches pitch for the remaining distance.

Rivets on End Connection.

The rivets fastening the stringers to the floor-beam are field rivets, and therefore the allowed intensities of stress in them must be reduced one third.

Two separate conditions affect the number of rivets at this end connection; first, the single shear on these rivets when one stringer has its maximum reaction, and secondly, the bearing on the web of the floor-beam, when the combined reactions at the two stringer ends resting on the floor-beam have their maximum value. For the first case, each rivet in single shear will carry $5410\times.8\times\frac{2}{3}=2885$ pounds, and with a maximum end reaction of 68,200 pounds, there will be required 24 rivets; in the second case, assuming the thickness of the floor-beam web plate at $\frac{9}{16}$ inch, the bearing capacity of one rivet will be $7380\times.8\times\frac{2}{3}=$ 3930 pounds; the simultaneous maximum combined reactions of the two stringers will be, for dead load 14,000

pounds, and for live load 84,000 pounds (see design of floor-beam for these reactions), or a total of 98,000 pounds; there will therefore be required 25 rivets. Thirteen rivets will however actually be placed in each 6×6 inch angle, making 26 rivets at this connection.

The bill of material with the weights for one stringer, and one half the cross bracing, using that indicated in Fig. 5, for which no actual design will be made, may be written as follows:

			Lbs. per Ft.	
1 web plate $32 \times \frac{7}{16}''$	29.1	feet long	@ 50.58	= 1470 lbs.
4 angles $6 \times 6 \times \frac{9}{16}''$	"	" "	@ 21.9	= 2550 "
2 cover-plates $14 \times \frac{9}{16}''$	19.25	" "	@ 26.78	= 1060 "
4 end stiffeners $6 \times 6 \times \frac{7}{16}''$	2.6	" "	@ 17.2	= 178 "
4 filling plates $6 \times \frac{9}{16}''$	1.6	" "	@ 11.48	= 74 "
$2\frac{1}{2}$ bracing angles $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}''$	9.5	" "	@ 2.1	= 50 "
About 450 pairs of $\frac{7}{8}''$ rivet-heads............			@ 22.2 lbs. per 100	= 200 "
			Total	= 5582 "

The actual weight per foot of stringer is thus seen to be 384 pounds, or 9 pounds per foot more than was assumed. This difference is so small as to require no correction in the design, but it is better to have the actual weight under rather than over that assumed.

Intermediate Floor-beams.

The maximum live-load floor-beam reaction occurs when wheel 5 is placed directly over the floor-beam. Wheels 1 to 4 will then be on one adjoining stringer and wheels 5 to 9 on the other, making the resulting reaction 84,000 pounds.

Assuming the dead weight of the floor-beam to be 300 pounds per linear foot, the dead-load reaction will become:

Dead weight of one stringer	= 11,300	pounds.
Dead weight of $\frac{1}{2}$ floor-beam	= 2,700	"
	14,000	pounds.

The sum of the dead- and live-load reactions will then be 98,000 pounds.

The stringers are spaced 7 feet 6 inches apart, and since the distance between trusses is 18 feet, the stringer is distant 63 inches from the centre line of the truss. Therefore the maximum bending of the floor-beam will be $98,000 \times 63 = 6,170,000$ inch-pounds. Assuming an effective depth of 42 inches, the flange area required will be $6,170,000/42 \times 10,000 = 14.69$ square inches. This section will be composed of 2 angles, $6 \times 4 \times \frac{1}{2}$ inches $= 9.6$ square inches gross $= 8.6$ square inches net; 1 cover-plate $14 \times \frac{1}{2}$ inches $= 7.0$ square inches gross $= 6.0$ square inches net. Total, 14.6 square inches net.

The cover-plate should have the following length:

$$l = 18\sqrt{\frac{6}{14.6}} = 11.5 \text{ feet};$$

owing to the details at the ends of the floor-beam the upper cover-plate will have a length of 12 feet 7 inches and the lower a length of 14 feet 9 inches.

On account of the large number of rivets made necessary at the end connection of the floor-beam, and the consequent weakening of the plate, the assumed intensity of shearing stress for the web has been taken at 4000 pounds per square inch of gross section. The area required will therefore be $98,000/4000 = 24.5$ square inches, and a plate $43 \times \frac{9}{16}$ inches $= 24.2$ square inches will fulfil the requirements. No intermediate stiffeners will be required, as the end connections of the stringers render end stiffeners unnecessary. The distance from back to back of the flange angles will be $43\frac{1}{4}$ inches. At the connection of the floor-beam to the post, a considerable portion of the web plate must be removed in order to leave room for the heads of the lower-chord eye-bars. It is therefore customary to cut

the web plate at some distance from the end, and to splice to it another plate of irregular shape having an equal cross-sectional area. The general detail drawing, Plate II, illustrates the method adopted in the present case. The central portion of the web is 10 feet 12 inches long and spliced to it at each end by means of two $\frac{7}{16}$-inch plates, placed over $\frac{1}{2}$-inch fillers, is another plate designated on the figure as "web 24 inches $\times \frac{9}{16}$ inch $\times$ 6 feet 3 inches long." To this end web plate are fastened the end stiffeners formed of two $6 \times 3\frac{1}{2} \times \frac{7}{16}$ inch angles. The end stiffener must be divided into an upper and lower part in order to allow the upper flange of the floor-beam to abut against the post. The lower portion of the end web plate will be cut away with rounded edges, to the form required by the disposition of the eye-bars, all floor-beams being made alike to conform to that post in which the greatest clearance is required. This edge will be protected by two angles $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inches, and 3 feet 6 inches long. The lower flange of the floor-beam will be connected to the foot of the post by a flat plate riveted to both, to provide against swaying, and to form part of the lower lateral wind system.

Riveting.

Instead of employing the rule used in finding the pitch of the rivets in the flanges of the stringers, it will be better to divide the amount of stress in the flanges of these beams between the post connection and the stringer connection, by the allowed stress capacity of one rivet; the quotient will determine the number of rivets between these two points. For example:

$$\frac{146,900}{7380 \times .8} = 25 \text{ rivets.}$$

These are easily provided for in the lower flange, but a pair of supplementary angles $6 \times 3\frac{1}{2} \times \frac{7}{16}$ inches and 3 feet $1\frac{1}{2}$ inches long must be fastened to the upper flange at its ends in order to provide for the requisite rivets to transfer the stress from the web plate to the flanges. The rivet design of this end connection is not very precise. The rivets piercing the spliced central portion of the web plate of the floor-beam really aid in carrying the stress through the splice-plate to the flanges. It is essential that all parts at this point shall be firmly and rigidly tied together. The pitch in the central portion of the beam will be 6 inches.

The number of rivets necessary on each side of the splice will be

$$\frac{98,000}{7380 \times .8} = 17,$$

and they will be placed in two rows, as shown in Plate II.

At least the same number of rivets will be necessary to fasten the end stiffeners to the web plate; but since 17 rivets cannot be placed in one row, the two rows shown on the plate will be inserted, even though all those there shown are not necessary. The resulting surplus may be considered as compensating for the tendency to pull on the rivet-heads, due to the deflection of the loaded beam.

The end stiffeners are connected to the post by field rivets. The thickness of the web of the post is three eighths inch, but this thickness will be increased by a diaphragm which will later be inserted between the channel webs, so that the single shear of the rivets determines their number.

The rivets required will therefore be $[98,000/(5410 \times .8 \times \frac{2}{3})] = 34$.

The rivets in the cover-plate will be pitched 3 inches for a distance of 15 inches from the ends of the cover-plate, and 6 inches for the remaining distance.

A small angle bracket consisting of a $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inch angle, 1 foot $\frac{1}{2}$ inch long, will be riveted to the floor-beam to form a seat for the ends of the stringer during erection. This bracket is used for temporary convenience only, and is not supposed to carry any stress after erection.

It is sometimes customary, if web plates have no splices, to take one sixth [or sometimes one eighth] of the web section as acting in either flange. If no rivet-holes were punched for the stiffeners, this method would be allowable. But such rivet-holes frequently take out considerable metal, and as the tension side of the plate only is affected, one sixth of the remaining metal ceases to be a proper proportion. On the whole, therefore, it is better to neglect the bending resistance of the web, and allow it to balance, so far as it may, the effect of the rivet-holes being out of the centre of gravity of the flange angles.

The bill of material and the weight of the floor-beam is given in the following table:

	Length.	Weight per Foot.	Total Weight, Pounds.
1 web plate $43 \times \frac{9}{16}''$	12′ 10″	82.24	1040
2 " plates $24 \times \frac{9}{16}''$	6′ 3″	45.90	573
2 flange angles $6 \times 4 \times \frac{1}{2}''$	16′ 10″	16.2	545
2 " " "	14′ 9″	16.2	478
1 cover-plate $14 \times \frac{1}{2}''$	12′ 7″	23.8	300
1 " " "	14′ 9″	23.8	350
4 end stiffening angles $6 \times 3\frac{1}{2} \times \frac{7}{16}''$	1′ $8\frac{3}{4}''$	13.5	90
4 " " " "	2′ 7″	13.5	140
4 " fillers $6 \times \frac{7}{16}''$	2′ 4″	8.93	83
4 " finishing angles $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}''$	3′ 6″	8.5	119
4 splice-plates $24 \times \frac{7}{16}''$	3′ 6″	35.70	500
4 filling plates $30 \times \frac{1}{2}''$	3′ $3\frac{1}{4}''$	51.00	660
4 upper flange, end angles $6 \times 3\frac{1}{2} \times \frac{7}{16}''$	3′ $1\frac{1}{2}''$	13.5	163
4 " " fillers $6 \times \frac{1}{2}''$	2′ $1\frac{1}{2}''$	10.20	86
2 " " " $3\frac{1}{2} \times \frac{9}{16}''$	1′ $1\frac{1}{2}''$	6.70	15
4 bracket angles $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}''$	1′ $0\frac{1}{2}''$	8.5	35
340 pairs of rivet-heads at 22.2 lbs. per 100.			150
			5327

The assumed weight is 5400 pounds, i.e., greater than the actual, as it should be.

Art. 13.—Specifications for Lateral and Wind Bracing.

The following specifications apply to the bracing:

§ *30. Allowed unit stress, in tension, for longitudinal, lateral, and sway bracing for wind and live-load stresses = 18,000 pounds per square inch.*

§ *31. Angles subject to direct tension must be connected by both legs, or the section of one leg only will be considered as effective.*

§ *33. Allowed unit stress, in compression, for lateral struts and rigid bracing* $= P = 13,000 - 70\, l/r$ *for wind stresses, where P, l, and r have the same meaning as previously. No compression member, however, shall have a length exceeding 125 times its least radius of gyration. Lateral struts, with adjustable bracing, will be proportioned by the above formula to resist the maximum due either to the wind and load or to an assumed initial stress of 10,000 pounds per square inch on all the rods attached to them.*

§ *35. Members subject to alternate stresses of tension and compression shall be proportioned to resist each kind of stress. Both of the stresses shall however be considered as increased by an amount equal to eight tenths of the least of the two stresses, for determining the sectional area by the above allowed unit stresses.*

§ *37. The rivets in the lateral and sway bracing will be allowed 50 per cent. increase over usual allowed unit stresses.*

§ *94. Where rods are used in the lateral, longitudinal, or sway bracing, they shall be square bars, but in no case shall they have a less area than one square inch. Rods with bent eyes must not be used.*

§ *95. All through bridges shall have latticed portals, of approved design, at each end of the span, connected rigidly*

to the end posts and top chords. They shall be as deep as the specified head room will allow.

§ 96. Where the height of the trusses exceeds 25 feet, an approved system of overhead diagonal bracings shall be attached to each post, and to the top lateral struts.

§ 97. All bars and rods in the web, lateral, longitudinal, or sway systems must be securely clamped at their intersections to prevent sagging and rattling.

Art. 14.—Design of Upper Longitudinal Wind Bracing.

The tension members will be formed of angles connected by both legs, so that the full area will become available. The following table shows clearly the method of design, the allowable unit stress being 18,000 pounds per square inch.

Member.	Stress in Pounds.	Area Required, Square Inches.	Section Adopted.	Gross Area of Section. in Sq. Ins.	Net Area of Section in Square Inches.	Rivets Required at End.
U_1U_2	+42,500	2.36	1–$3\frac{1}{2}\times3\frac{1}{2}\times\frac{7}{16}''$ angle	2.88	2.50	6
U_2U_3	+33,000	1.84	1–" × " × $\frac{3}{8}''$ "	2.50	2.17	5
U_3U_4	+23,700	1.33	1–" × " × " "	2.50	2.17	4
U_4U_5	+14,300	0.80	1–" × " × $\frac{5}{16}''$ "	2.09	1.81	2
U_5U_6	+ 4,700	0.28	1–" × " × " "	2.09	1.81	1

These members are connected to the upper chord by $\frac{3}{8}$-inch gusset-plates. The bearing capacity of one machine-driven rivet in this thickness of plate will therefore be 4920×150 per cent. $=7380$ pounds, and this quantity determines the number of rivets required at the end connections as given in column 7, although all the connections will be made uniform with six rivets. The compression members at right angles to the axis of the bridge are not designed by any exact theory. They will be formed either of cross frames or struts, as indicated on the stress sheet, and they are more than ample in section to sustain the direct compressive stresses there indicated; in fact, since their main

duty is to stiffen the structure and to cause the two trusses to act as a whole, they are made as rigid as the judgment of the engineer requires. The portal bracing is designed in a somewhat similar way, although tested by computations in its various parts.

The struts will be placed at those points, U_4 and U_6 where the upper chord is continued in a straight line, and where there are no inclined web members. They will be composed of four angles, each $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}$ inches, built in I form with double lattice bars $2\frac{1}{4} \times \frac{3}{8}$ inches, and connected to two gusset-plates, one lying on top of the cover-plate of the upper chord and the other fastened to the bottom angle of the upper chord.

The cross frames will be placed at all the other panel points. They will be formed of an upper strut, like that just described, and a lower strut placed at the upper clearance limit and formed of four angles each $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}$ inches, latticed together in I form by bars $2\frac{1}{4} \times \frac{5}{16}$ inches. The lower strut will be fastened to the post by means of a $\frac{5}{16}$-inch gusset-plate lying between two $3\frac{1}{2} \times 3 \times \frac{5}{16}$ inch angles, two feet long, the latter being riveted to the centre of the posts by twelve rivets. Between the two I struts will then be placed a compound lattice-work composed of single $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{16}$ inch angles tied together at intersections by small square plates with five rivets at each crossing and fastened to the struts by $\frac{5}{16}$-inch gusset-plates, the details of which are shown in Plate II. The cross frames are really the intermediate transverse bracing, the theory of whose design would be determinate if it were not the duty of the upper horizontal lateral bracing to transfer all the wind load to the upper ends of the end posts. It is possible to design the cross frames by ignoring this upper wind bracing. Their main function, however, is to stiffen the bridge under rapidly moving train loads, rather than to give additional stability against wind loads.

Art. 15.—Design of Portal Bracing.

The portal bracing will be made as rigid as possible and will take the form of a latticed girder placed in the plane of the pins of the end posts, as shown in outline on the stress sheet. The upper and lower flanges will be made alike of two $6 \times 4 \times \frac{3}{8}$ inch angles with a $14 \times \frac{3}{8}$ inch web plate. The two flanges will be tied together by a compound latticing composed of single $5 \times 3\frac{1}{2} \times \frac{3}{8}$ inch angles. The upper flange will be fastened to the cover-plate of the top chord, U_1U_2, by means of a bent gusset-plate, as shown in Plate II; the lower flange will be fastened to the side plate of the end post in a manner similar to that of the cross frames. The lower flange will, moreover, be strengthened

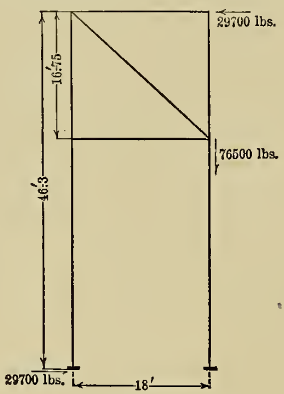


FIG. 6.

at each end connection by means of a plate and angle bracket, composed of a $\frac{3}{8}$-inch plate and $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inch angles, as shown in Plate II. The outer end of the bracket will be supported from the latticing of the portal frame by a single $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$ inch angle.

The wind force acting at the upper end of the end post is 29,700 pounds. The length of the end post is 46.3 feet

and the distance between trusses 18 feet. The increase of load on one truss, which acts as if hung at the connection point of the lower flange of the portal, will therefore be 76,500 pounds. The multiple bracing employed produces ambiguity in the stresses of the portal members. If the assumption be made, as shown in the accompanying figure, that the multiple bracing is replaced by a single tension-bar, and that the wind reaction is entirely resisted at the foot of the leeward post, the stress in the lower flange will be: $\frac{76,500 \times 18}{16.75} = 82,200$ pounds compression. The allowed intensity of stress is $P = 13,000 - 70l/r$, in which $l = 216$ inches and r may be taken approximately as 3 inches. Then $P = 8000$ pounds per square inch, approximately, and the sectional area required will be $\frac{82,000}{8000} = 10.4$ square inches.

The actual area will be:

2	$6 \times 4 \times \frac{3}{8}$ inch angles	$= 7.22$	square inches.
1	$14 \times \frac{3}{8}$ inch plate	$= 5.25$	" "
		12.47	" "

The section adopted is therefore sufficient.

The stress in the upper flange may be taken to be equal to the wind reaction transmitted from one truss to the other; its section, although so taken, need not be as great as that of the lower flange.

Art. 16.—Design of Lower Longitudinal Wind Bracing.

Although the inclined web members in this horizontal truss may suffer apparently reversal of stress, it must be remembered that the two systems of bracing provide for opposite directions of the wind without such reversal.

The specifications in regard to reversal of stresses do not therefore become operative. If it be assumed that the inclined web members act in compression, they must be designed with the prescribed unit compressive stresses. As they are fastened at two points to the stringers and at the centre to each other, it may be proper also to assume the effective length of a member to be the distance from its end to the point of intersection with the centre line of the stringer, i.e., in this case about 120 inches. These members will be formed of two unequal legged angles, with the longer legs vertical and riveted back to back. The following table shows the sections adopted:

Member.	Stress in Pounds, Compression.	Radius of Gyration in Inches, Final Section.	Allowed Unit Stress in Pounds per Square In.	Area Required in Square Inches.	Section Adopted. 2 Angles.	Actual Section in Square Inches.	Number of Rivets Required at End Connection.
L_0L_1	71,600	1.56	7610	9.4	$5\times4\times\frac{9}{16}''$	9.52	14
L_1L_2	59,200	1.56	7610	7.8	" $\times\frac{1}{2}''$	8.58	11
L_2L_3	47,700	1.56	7610	6.3	" $\times\frac{3}{8}''$	6.46	10
L_3L_4	36,900	1.56	7610	4.85	" $\times\frac{3}{8}''$	6.46	7
L_4L_5	26,800	1.23	6170	4.35	$4\times4\times\frac{3}{8}''$	5.7	6
L_5L_6	17,500	1.23	6170	2.84	"	5.7	4

It will be seen that the sections adopted for the panels near the centre of span are greater than actually needed; but since a portion of the duty of these members is to stiffen the structure under rapidly moving loads their section may properly be greater than required by the computed stresses.

The connection of these members to the posts is made by a flat plate $\frac{1}{2}$ inch thick, which in turn is fastened to the foot of the post by $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$ inch angles, as shown in Plate II. The amount of stress which these angles must carry to the plate is the difference of wind stresses found in the two adjacent lower chord panels which has a maximum value of $222,000-122,000=100,000$ pounds at the foot of U_1L_1.

The specifications for wind-bracing riveting show the single shearing capacity of one rivet to be 5410×150 per cent. $\times \frac{2}{3} = 5410$ pounds and the bearing capacity in a $\frac{5}{8}$-inch thickness of metal to be 4920×150 per cent. $\times \frac{2}{3} = 4920$ pounds. At the foot of U_1L_1 there will therefore be required $\frac{100,000}{4920} = 21$ rivets; since the two angles inside the post do not afford space for all these rivets, a U-shaped plate will be riveted within the post, which will permit placing the required number of rivets. At the foot of U_2L_2, the maximum stresses carried by the plate will be 78,000 pounds, and will require 16 rivets. At the foot of U_3L_3 the stress will be 56,000 pounds, and 12 rivets are required. Plate II shows the disposition of these rivets in the angles. The number of rivets connecting the ends of the lateral bracing to the plate, as given in column 8 of the above table, was found in the same way as for the upper chord.

The lower flanges of the floor-beams act as struts in the lower lateral system. The $\frac{1}{2}$-inch plate which is used as a connection at the foot of the posts is sufficiently extended to be firmly riveted to the lower flange of the beam. The number of rivets necessary may be determined by assuming that the plate transmits to the floor-beam that component of the stress in the inclined lateral members which acts in a direction at right angles to the axis of the chords.

Art. 17.—Design of Bedplates, Friction-rollers, and Pedestals.

§ 100. All bedplates must be of such dimensions that the greatest pressure upon the masonry shall not exceed 250 pounds per square inch.

§ 101. All bridges over 75 feet span shall have at one end nests of turned friction-rollers running between planed surfaces. These rollers shall not be less than $2\frac{7}{8}$ inches diameter

for spans 100 feet or less, and for greater spans this diameter shall be increased in proportion of 1 inch for 100 feet additional.

The rollers shall be so proportioned that the pressure per linear inch shall not exceed the product of the diameter in inches by 300 pounds.

§ 104. Pedestals shall be made of riveted plates and angles. All bearing surfaces of the base-plates and vertical webs must be planed. The vertical webs must be secured to the web by angles having two rows of rivets in the vertical legs. No base-plate or web connecting angle shall be less in thickness than $\frac{3}{4}$ inch. The vertical webs shall be of sufficient height, and must contain material and rivets enough to practically distribute the loads over the bearings or rollers.

Where the size of the pedestal permits the vertical webs must be rigidly connected transversely.

§ 105. All the bedplates and bearings under fixed and movable ends must be fox bolted to the masonry; for trusses, these bolts must not be less than $1\frac{1}{4}$ inches diameter.

The dead-load reaction at the base-plate is 325,000 pounds and the live load 420,000 pounds, a total of 745,000 pounds. The masonry bearing will be $\frac{745,000}{250} =$ 2980 square inches. The diameter of the rollers according to the specification should be at least $2\frac{7}{8} + \frac{250}{100} = 5\frac{3}{8}$ inches, but since it is better practice to build segmental rollers the diameter of these rollers may be taken at 8 inches. Assuming then that eight rollers will furnish a proper form of distribution, the number of linear inches of length per roller required will be $\frac{745,000}{8 \times 300 \times 8} = 38.9$ inches; the rockers will therefore be 8 in number, 8 inches high, 5 inches wide, and 4 feet 5 inches long, rolling on a base plate $1\frac{1}{4}$ inches thick, 4 feet 2 inches $\times$ 5 feet 6 inches in area. A $1\frac{1}{4}$-inch

plate, 50 inches $\times$ 4 feet 6 inches, placed over the rockers will form the seat or bedplate for the pedestal proper. Both base-plate and bedplate will have riveted to them, so placed as to be directly below the centre of the end post, a flat bar, $2\frac{1}{4}$ inches $\times$ $\frac{3}{8}$ inch $\times$ 4 feet 2 inches long, made to fit corresponding grooves turned in the rockers to prevent all lateral motion of the rockers or pedestal. The rollers will be fastened rigidly together in their proper positions with $\frac{3}{8}$-inch clear spacing by a $\frac{3}{4}$-inch strap on each side 2 feet 3 inches long secured to the axis of each roller by a nut.

The pedestal must distribute uniformly to the shoe or bedplate the vertical load carried by the pin at the foot of the end post. For this reason it should be made deep enough to prevent appreciable deflection, but not so deep as to cause a tendency to buckle the side-plates. A completely rational design for a pedestal is practically impossible. It is treated in some cases as a short plate girder uniformly supported along the bottom and carrying a concentrated load at its centre. It is more usual to select some tried form of pedestal and adapt it for use in accordance with good judgment; this will be done in the present case. The side-plates of the pedestal supporting the end pin must first be considered. The diameter of this pin should therefore be known, and in this instance may be assumed to be 7 inches. With an allowable bearing pressure of 15,000 pounds per square inch, the total thickness of the side-plates for one half of one pedestal will be $745{,}000/2 \div (15{,}000 \times 7) = 3.55$ inches. If the main side-plates are fastened between two $6 \times 6 \times \frac{3}{4}$ inch angles, $\frac{3}{4}$ filling-plates will be required above the angles. In addition to these two $\frac{3}{4}$-inch plates there will then be needed two $\frac{11}{16}$-inch plates and one $\frac{3}{4}$-inch plate, giving a total thickness of $3\frac{5}{8}$ inches. An additional plate $\frac{7}{16}$ inch thick, called a lap-plate, projecting upward within the

end post, will be fastened to the inside of these plates. The riveting of these connections is clearly shown in Plate II. It will be seen that a $\frac{7}{16}$-inch cover-plate and two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ inch angles are added to the upper edges of the pedestal in order to make it conform in finished appearance to the end post.

The pedestal at the fixed end of the truss will be exactly similar to the one above described, but the rollers will be dispensed with and the difference in height will be provided for in the pedestal itself.

The base-plate, as shown in the figure, will be rigidly secured to the masonry by $1\frac{1}{2}$-inch anchor-bolts 2 feet long. On the outer side of the rollers, a dust-guard of two angles and a plate built in Z shape will be connected to the anchor-bolts.

Art. 18.—Design of End Floor-beam.

The end floor-beam, which is fastened to the shoe-plate and which carries not only the end stringers, but also the short cantilever-brackets supporting the track from the abutment itself to the end stringer, still remains to be designed.

The end shear of this floor-beam will be the live-load shear of a single stringer, 62,500 pounds, together with an estimated dead-load shear of 7500 pounds, or a total of 70,000 pounds. The maximum moment will then be 70,000 pounds $\times$ 63 inches = 4,440,000 inch-pounds. Assuming the effective depth of girder to be 54 inches, the required sectional area of one flange will be 8.15 square inches, and it may be composed of two angles $6 \times 4 \times \frac{1}{2}$ inch with a gross section of 9.6 square inches and a net section of 8.6 square inches.

The web will be a $55\frac{3}{4} \times \frac{3}{8}$ inch plate with an area of 20.9 square inches. Allowing a compressive intensity

of stress of 8000 pounds per square inch in the end stiffeners, the area required will be 8.75 square inches. Two $6 \times 4 \times \frac{1}{2}$ inch angles having an area of 9.5 square inches will be used, with fillers.

The end floor-beam rests on and is riveted to a horizontal $\frac{1}{2}$-inch plate, which in turn is solidly united to the shoe-plate. To prevent lateral deflection, the upper flange of this floor-beam is tied to the end post by means of a horizontal bent plate, and in addition a vertical triangular-shaped plate is fastened to the end stiffeners and to the shoe-plates, as shown clearly by Plate II. The plane of the lower lateral bracing does not coincide with the plane of the shoe-plate, and to provide a proper connection, two $6 \times 4 \times \frac{1}{2}$ inch angles are riveted to the web of the floor-beam at the proper elevation.

The end-stringer bracket is also shown in sufficient detail in Plate II. The floor-beam is slotted in order to allow the tie-plate forming the cover-plate of the bracket to pass through it. This tie-plate is riveted to the upper flange of the adjacent intermediate stringer, to relieve as much as possible the direct stress of tension which may come upon the rivet-heads connecting the bracket to the floor-beam.

Art. 19.—Design of Pins and Joint Details.

§ 38. Pins shall be proportioned so that the shearing stress shall not exceed 9000 pounds per square inch; nor the crushing stress on the projected area of the semi-intrados of any member (other than forged eye-bars, see § 80) connected to the pin be greater per square inch than 15,000 pounds; nor the bending stress exceed 18,000 pounds, when the applied forces are considered as uniformly distributed over the middle half of the bearing of each member.

§ 78. The lower chord shall be packed as narrow as possible.

§ 80. The diameter of the pin shall not be less than $\frac{3}{4}$ the largest dimension of any eye-bar attached to it. The several members attaching to the pin shall be so packed as to produce the least bending moment upon the pin, and all vacant spaces must be filled with wrought filling rings.*

§ 88. Where necessary, pinholes shall be reinforced by plates, some of which must be of the full width of the member, so that the allowed pressure on the pins shall not be exceeded, and so that the stresses shall be properly distributed over the full cross-section of the members. These reinforcing plates must contain enough rivets to transfer their proportion of the bearing pressure, and at least one plate on each side shall extend not less than 6 inches beyond the edge of the bottom plates (§ 87).

§ 87. The open sides of all compression members shall be stayed by batten plates at the ends. . . . The batten plates must be placed as near the ends as practicable, and shall have a length not less than the greatest width of the member, or $1\frac{1}{2}$ times its least width.

§ 89. Where the ends of compression members are forked to connect to the pins, the aggregate compressive strength of these forked ends must equal the compressive strength of the body of the members.

In addition to these direct specifications the following may be noted:

§ 60. In compression members, abutting joints with

---

* The following analysis furnishes the foundation for this specification. If the width of an eye-bar be represented by w, and the depth by h (the thickness of the head is taken the same as that of the bar), and if T be the permissible intensity of tensile stress, the total stress carried by the bar will be $w \cdot h \cdot T$. If the diameter of the pin be represented by d and C be the permissible bearing intensity of stress on the pin, then $d \cdot w \cdot C$ will represent the bearing allowed on the pin. The two quantities $w \cdot h \cdot T$ and $d \cdot w \cdot C$ should be equal; if the ratio of C/T be taken at $4/3$, then $whT = dwC$ and $d = 3/4h$.

planed faces must be sufficiently spliced to maintain the parts accurately in contact against all tendencies to displacement.

§ 61. In compression members, abutting joints with untooled faces must be fully spliced, as no reliance will be placed on such abutting joints. . . .

The bending in pins and the allowed bearing of the plates on pins are peculiarly related; the larger the diameter of the pin, the narrower are the necessary bearing surfaces and consequently the smaller are the bending moments. If the diameter of the pin be decreased, the bearing surfaces must be increased in width. In that case the bending moments are increased and the pin diameter may have to be increased. It will usually be impossible to obtain an economical balance for these conflicting conditions throughout the truss without many different sizes of pin. It is therefore customary to fix upon one size of pin for the lower chord, and another smaller one for the upper chord, then test them for bending and design the bearing surfaces to correspond with the diameters so chosen. The packing or arranging of the eye-bars must be so chosen as to reduce the bending to a safe minimum.

Specification § 80 at once fixes a minimum limit for the size of the lower chord pins and also of the pins at the upper extremity of the end post at 6 inches; and the pins in the upper chord at $5\frac{1}{4}$ inches. The sizes tentatively assumed will be 7 inches for the first set of pins and 6 inches for the latter. The bearings of the various members for these sizes of pins are then designed, and then all the members at any one point so packed as to produce the least practicable bending moment. The "play" or spacing allowed in packing eye-bar heads may sometimes be taken

as small as $\frac{1}{32}$ inch to allow for slight imperfections in manufacture, but it is usually taken as $\frac{1}{8}$ inch for each eye-bar head. This amount may be increased to meet the requirements of any special case. The clear distance between built members such as chords or posts is usually greater; it may be taken as high as 1 inch. It will be found necessary at some connecting points to cut away parts of the flanges of channels or angles in built-up members on account of the interference of inclined members. If this must be done, the remaining part of the member must always be tested by computation as to its strength, considering it as a short piece in compression. In most cases the usual pin-bearing plates are sufficient to replace the metal cut away. The following formula has been much used for the purpose of designing such forked portions of the ends of posts:

$$t=\frac{P}{8000b}+\frac{l}{27}, \quad \ldots \ldots \ldots \quad (1)$$

in which t represents the total thickness of metal whose width is b; P the total load on one jaw of the post (usually one half the total load carried by the post or column) and l the distance from the centre of the pinhole to the last centre line of rivets in the body of the column back of the cut in the angle or in the flange of the channel. This formula is applicable to steel with ultimate tensile resistance running from 60,000 to 68,000 pounds per square inch. For higher steel, or for highway bridges, or for other structures where less margin of safety may be justifiable, the value of t may be made correspondingly less than given by eq. (1).

Pin-plates in Posts.

The posts at their lower extremities carry to the pins not only their maximum stresses as members of the truss, but also the compression due to their acting as supporting struts for the floor-beams. It is possible and it should be assumed that the maximum post stress and the greatest panel load occur together. In the present case this requires that the lower ends of the posts be reinforced with sufficient thickness of bearing plates to carry 98,000 pounds in addition to the stresses taken from the stress sheet (Plate I). In order that this floor-beam load may be distributed uniformly to the pin by both jaws of the column, it is necessary that the two sides of the post be riveted firmly together to act as a whole; otherwise the inside jaw will carry the greater portion of the load. This is usually accomplished by inserting between the two parts of the column, at the end of the floor-beam, a diaphragm which acts as a continuation of the web plate of the beam. This diaphragm will be composed of a $\frac{3}{8}$-inch plate riveted between four $4 \times 3 \times \frac{3}{8}$ inch angles. The method of fastening is clearly shown in Plate II.

Member L_1U_1—Lower End.

This member needs to be fastened at its lower end only to increase the general stiffness of the structure, since it is simply a hanger to carry load to the upper panel point. The U-shaped plate to which the lower lateral bracing is attached furnishes inside pin-plates, but in addition two $\frac{3}{8}$-inch plates will be fastened to the outside extending upward to the bottom of the floor-beam, so that the latter may have support during erection.

Member L_1U_1—Upper End.

A 1-inch plate, bearing on a 7-inch pin, may carry 105,000 pounds. The load to be carried at this point is 117,000 pounds; therefore 1.12 inch total width of bearing will be required. As the webs of the channel are each 0.375 inch thick, the thickness of pin-plate required would be 0.37 inch, but since the net section of the channels is reduced by the diameter of the pin, it will be advisable to use two $\frac{5}{8}$-inch pin-plates, especially as the vertical distance from the top of the pin to the upper extremity of the channels is but 6 inches. The net section through this point should not be less than two thirds of the transverse net section, and the $\frac{5}{8}$-inch pin-plates chosen will satisfy the condition.

Since the allowable stress in the main section is 6000 pounds per square inch, each pin-plate may be supposed to carry $15 \times \frac{5}{8} \times 6000 = 56,300$ pounds, and as the rivet bearing in a $\frac{5}{8}$-inch plate is 4920 pounds, twelve rivets will be required.

Member U_2L_2—Lower End.

The sum of the dead and live loads carried at this point, together with the floor-beam load of 98,000 pounds, may produce a total load of 357,000 pounds, or 178,500 pounds on each jaw of the post resting on a 7-inch pin. A total thickness of $\frac{178,500}{105,000} = 1.7$ inches will therefore be required. A $\frac{5}{8}$-inch inside, and a $\frac{5}{8}$-inch outside pin-plate, together with the $\frac{9}{16}$-inch main plate, give a thickness of 1.81 inches, and will prove satisfactory.

Since the bearing value for one rivet in a $\frac{9}{16}$-inch plate is 7380 pounds, or less than the double shear value, the former quantity will determine the number of rivets. It

may be assumed that the total stresses carried by the different plates are proportional to their thickness; therefore $(\frac{5}{8}+\frac{5}{8}) \div 1\frac{9}{16} = 69$ per cent. of the total stress must be transferred by rivets from the main plate to the pin-plates. In this case $0.69 \times 178,500 = 123,000$ pounds; therefore at least seventeen rivets will be required; Plate II shows the arrangement of plates and rivets.

Member U_2L_2—Upper End.

The 6-inch pin at this point will carry a bearing pressure of 90,000 pounds per inch width of plate, the load on each jaw of this post being 129,500 pounds. 1.44 inches thickness of bearing will be required, and this thickness will be afforded by the $\frac{9}{16}$-inch main plate, with a $\frac{7}{16}$-inch outside and a $\frac{1}{2}$-inch inside pin-plate. The bearing value of one rivet in a $\frac{9}{16}$-inch plate is 7380 pounds, and the rivets transfer 63 per cent. of $129,500 = 81,500$ pounds, to the pin-plates, hence the required number will be 11. Plate II shows the arrangement adopted.

Member U_3L_3—Lower End.

The load carried by one jaw resting on the 7-inch pin, including the floor-beam load, will be 101,500 pounds, and the thickness required will be $\frac{101500}{105000} = 0.97$ inch. A $\frac{5}{8}$-inch pin-plate in addition to the $\frac{3}{8}$-inch web of the channel will therefore suffice. The stress transferred to the pin-plate will be 63,500 pounds, and as the bearing value of one rivet in a $\frac{3}{8}$-inch plate is 4920 pounds, thirteen rivets will be required.

Member U_3L_3—Upper End.

The thickness of plates for one jaw bearing on a 6-inch pin will be $\frac{52500}{90000} = 0.58$ inch. A $\frac{1}{2}$-inch pin-plate will be

riveted to the outside of the channel and the number of rivets required will be 6.

It will not be necessary to repeat these operations in detail for the other posts of this truss, the method of design being precisely the same as those already given.

Art. 20.—Design of Pin Connections of Upper Chord Members.

As the joint of two adjoining upper chord members having different inclinations is always at the angle in the chord with its plane passing through the axis of the pin, it is usually made abutting and riveted so as to make the chord continuous. This makes the chord stiffer than if the joints were pin-bearing. It has become almost the invariable practice, however, to make the joint at the top of the end post an open pin-bearing one. Since all abutting faces will be planed, specification § 60 requires no splice-plates except those sufficient to prevent displacement. The only stress transferred from a pin to a chord member will then be the resultant, in the direction of the chord member, of the maximum stresses in the web members meeting at that point, i.e., the increment of stress. Sufficient pin-bearing area must be provided to carry this increment of stress.

The joint for adjoining chord members whose axes are in the same straight line is never made at a pin, but at some distance from it, on that side away from the centre. This procedure is adopted for convenience in erection, which usually proceeds from the centre towards the ends. This joint is placed as near the pin as practicable, and in a chord 20 inches deep, it is usual to have three rows of rivets in the splice-plate on each side of the joint; for splices in chords 24 to 30 inches deep four rows are usually inserted. At such a joint a batten plate on the bottom and a short cover-plate on top are used to strengthen the splice.

It has already been noted that the axes of the pins have been placed $14\frac{1}{2}$ inches below the edge of the cover-plate, or 0.7 inch below the centre of gravity of the chord section. This tends in some degree to neutralize the flexure due to own weight by the counterflexure introduced by the increment of chord stress.

In the bridge under consideration no joint will be made at panel points U_4 and U_6; the members $U_5U_4U_5$ and $U_5U_6U_5$ will each be made in single pieces about 59 feet long, which is entirely practicable. The supporting struts which frame into these members at their centre points are connected by means of angles, as shown in Plate II. The joint for U_2 is made 2 feet $4\frac{1}{2}$ inches away from the centre of U_2 and $\frac{7}{16}$-inch splice-plates will be riveted on the inside of the side-plates of the chord, with four rows of rivets on each side of the joint. A $\frac{1}{8}$-inch filler is necessary to allow for the difference in thickness of plates in the two chord members. The $\frac{1}{2}$-inch outside plate will be continued to cover the joint.

The maximum increment of stress in U_2U_3 given by the web member at U_2 is 140,000 pounds, or 70,000 pounds for each jaw. The 6-inch pin can transfer this stress to the $\frac{13}{16}$-inch side-plate, but for increased stiffness and solidity a $\frac{1}{2}$-inch plate with $\frac{1}{2}$-inch filler will be riveted on the outside of each jaw. As may be observed, all connections are made so strong that a member should fail rather in its body than at its end connections.

Connection at U_3.

The maximum increment of stress for the member U_3U_4 at panel U_3 in the direction of the axis of U_3U_4 may be found graphically from the stresses in the web members to be 200,000 pounds. The thickness of each jaw bearing on a 6-inch pin must then be at least 1.1 inches. It will

be seen in Fig. 7 that besides the $\frac{9}{16}$-inch filler-plate and the $\frac{3}{8}$-inch lap-plate, a $\frac{3}{8}$-inch inside and a $\frac{3}{8}$-inch outside pin-plate have been used. This is more than the computations require, but not more than desirable for the stiffness of the continuous upper chord.

Plates corresponding to those riveted to U_3U_4 are fastened to U_2U_3 at U_3. The outside edges of the main side-plates of adjacent members should always be in line. In this truss the uniform distance from back to back of

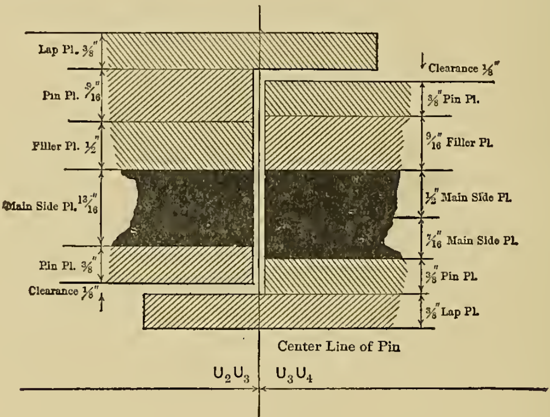


FIG. 7.

these plates is 23 inches. The abutting pieces at U_3 are shown in Fig. 7.

The stress carried by each plate will be determined by multiplying its thickness in inches by the allowed bearing of a 1-inch plate on a 6-inch pin, and this quantity determines the number of rivets required in each plate. These rivets must be designed not only for single shear for each separate plate, but also for the combined bearing of the outside plates on the main inside web plate. Where field rivets are used their stress value is only two thirds that of shop rivets.

If the thickness of pin-plates is large, considerable

bending, indeterminate in amount, may take place in the rivets, and it is proper on this account to add more rivets than those actually required by computation. Moreover, since the cover-plate and top angles of the chord section receive their stress cumulatively, the pin-plates which transfer a part of the stress to them should be made long enough to enable this stress to be transferred directly, instead of first passing through the main web plate.

Since there is thus a certain amount of indetermination in the transmission of stress in an abutting joint, such a joint is not designed strictly according to computations, but more or less according to experience tempered by the engineer's judgment. The design adopted must, however, always be tested by some method of analysis to determine its safety.

Connection at U_5.

The pin-plates at this panel point are designed in a manner similar to that followed for U_3. The design adopted, Plate II, provides abundant material.

Art. 21.—Design of Details at Ends of End Post.

Lower Extremity.

The end connections of L_0U_1 are not abutting, but pin-bearing. Hence all the stress in L_0U_1, 899,000 pounds, is transferred to the pin and from that to the pedestal plates.

A 1-inch plate bearing on a 7-inch pin can carry $7 \times 15,000 = 105,000$ pounds; therefore a 4.28-inch thickness of pin-plate is required for each jaw. There will then be added to the main web plates $\frac{1}{2}$ inch and $\frac{7}{16}$ inch thick respectively, an outside $\frac{5}{8}$-inch lap-plate, a $\frac{5}{8}$-inch plate below this, and a $\frac{9}{16}$-inch plate over a $\frac{1}{2}$-inch filler. On the inside, next to the side-plate, there will be a $\frac{9}{16}$-inch

plate, and then a $\frac{7}{16}$-inch plate; this gives a bearing thickness of $4\frac{1}{2}$ inches. The following rivets will be required in single shear for the various plates; Plate II shows their distribution:

Outside Plates.		Inside Plates.	
$\frac{5}{8}$-inch lap-plate.	12 rivets	$\frac{7}{16}$-inch plate.	9 rivets
$\frac{5}{8}$-inch plate.	12 "		
$\frac{9}{16}$-inch plate.	11 "	$\frac{9}{16}$-inch plate.	11 rivets
$\frac{1}{2}$-inch plate.	10 "		

Upper Extremity.

The same thickness of $4\frac{1}{4}$ inches of pin-plate is required at the upper extremity as at the lower, but a portion of this thickness will be formed of an inside diaphragm, partly to avoid the inside pin-plates interfering with the web members, and partly for the sake of the additional stiffness gained by this form of connection.

The diaphragm will be composed of a $\frac{11}{16}$-inch web, with four $4 \times 3\frac{1}{2} \times \frac{1}{2}$ inch angles, built in the form of a vertical I. Two $\frac{1}{2}$-inch fillers and two $\frac{9}{16}$-inch plates will be added to the diaphragm to bear on the pin. Fig. 8 shows clearly the dimensions of the pin-plates fastened to the side-plates. The computations need not be repeated here, it being remembered that one half the diaphragm plate thickness should be included in the thickness of one jaw.

End Connection of U_1U_2.

The connection of U_1U_2 at U_1 is designed in precisely the same manner as the end post connection at U_1. The stress in one jaw is 429,000 pounds, and 4.1 inches is the thickness of pin-plates required. An inside diaphragm will again be used, and Fig. 8 shows clearly the sections adopted.

Since the end post and U_1U_2 are not riveted together it is usual to allow a small clearance between them for ease in erection. Fig. 8 shows the clearance adopted.

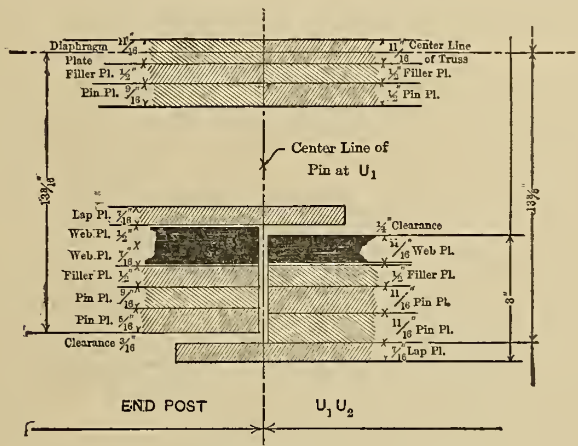


FIG. 8.

It will be seen that open joints are subject to a better defined analysis than abutting joints, but they form a heavier, more cumbersome, and less rigid connection.

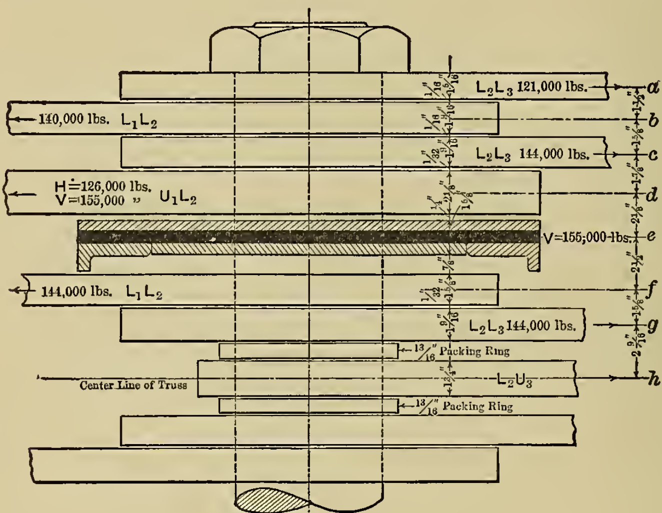
Art. 22.—Bending of Pins.

Lower Chord Packing.

Panel Point L_2.

The packing of pin L_2 and the computations of the bending moments in it will furnish characteristic operations typical of all other cases. Fig. 9 shows the disposition of the members meeting at L_2, the clearances, and distances between centres of adjacent pieces and the stresses carried by the pieces when L_1L_2 has its maximum

stress. The maximum stresses in all the members meeting at one point are not concurrent, since they are caused by different positions of the moving load. It becomes necessary to test each pin (1) with the maximum stresses existing in the web members, and (2) with the maximum stresses existing in the chord members. (It is sufficiently accurate to assume that the same position of loading causes the maximum stresses in the two adjacent chord members or in the web members converging on the pin.)



7" PIN AT L_2

FIG. 9.

As a rule, the greatest pin-bending in the lower chord occurs with the greatest chord stresses, and in the upper chord with the greatest web stresses.

The wind load stresses in the lower chord may be neglected as a factor in producing bending in the pins.

Taking moments about the centre of each piece in Fig. 9 in turn there is found:

	Horizontal Moment.	Vertical Moment.
About b	+181,500 inch-pounds	
" c	+151,000 "	
" d	+385,000 "	
" e	+383,000 "	330,000 inch-pounds
" f	+380,000 "	330,000 "
" g	+144,000 "	

The maximum moment, therefore, occurs at e and is equal to

$$\sqrt{\overline{383,000}^2 + \overline{330,000}^2} = 505,000 \text{ inch-pounds.}$$

The allowed unit bending stress is 18,000 pounds per square inch, and the bending resistance of a 7-inch pin is 606,100 inch-pounds. The packing at this point is, therefore, satisfactory.

If the bending be computed when L_2U_1 has its maximum stress, the vertical component of which is 166,000 pounds, it is seen that the vertical moment would be increased slightly, but the horizontal moment would be more than correspondingly decreased. The maximum bending moment on the pin occurs in this case, therefore, with the maximum chord stress.

Other panel points in the lower chord are treated in precisely the same manner· Plate II shows the packings employed.

Upper Chord Packing.

Panel Point U_3.

The packing at panel point U_3 may be assumed characteristic of the entire upper chord; Fig. 10 shows clearly the spacing and also the stresses in the different parts, when U_3M_4 takes its maximum stress. Since the two chord members U_2U_3 and U_3U_4 are abutting and as there

is practically no eccentricity between them, the resultant stress in the web members U_3L_3 and U_3M_4 is entirely resisted by U_3U_4, the horizontal and vertical components of which are easily found graphically.

The maximum stress in U_3M_4 is $+294,000$ pounds, and the concurrent stress in U_3L_3 is $-86,000$ pounds.

The side-plates of the chord form the supports of the ends of the pin, the centre of the supporting forces being

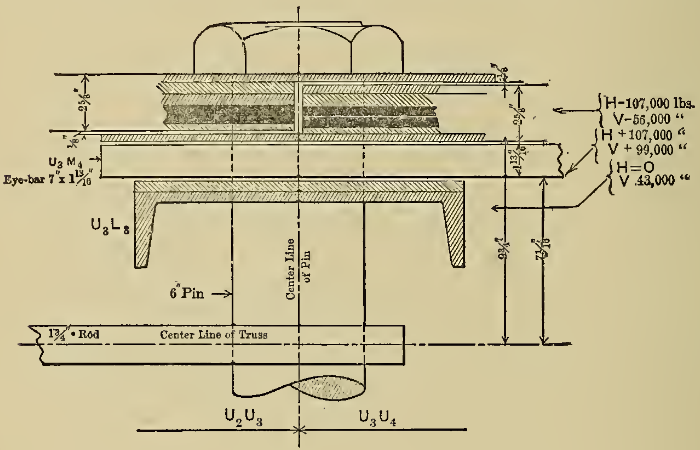


FIG. 10.

at the centre of the plates, as shown in Fig. 10. The members causing the bending in the pin are U_3M_4 and U_3L_3, the stress in the latter, $-86,000$ pounds, being wholly vertical. The horizontal and vertical components of the stress in one half of U_3M_4 are:

$$\text{H.} = +107,000 \text{ pounds}$$
$$\text{V.} = +\ \ 99,000 \quad \text{"}$$

Fig. 10 shows that the bending moment in the central portion of the pin is then composed of

$$\text{Hor. mom.} = 107,000 \times 2\tfrac{1}{16} = 221,000 \text{ inch-pounds}$$
$$\text{Vert. " } = 99,000 \times 2\tfrac{1}{16} - 43,000 \times 3\tfrac{5}{8} = 48,000 \text{ inch-pounds}$$

Hence the resultant moment is

$$\sqrt{221{,}000^2 + 48{,}000^2} = 226{,}000 \text{ inch-pounds.}$$

The allowed bending resistance of a 6-inch pin is 318,000 inch-pounds. A 6-inch pin is therefore satisfactory at U_3.

Other upper chord panel points are investigated in exactly the same manner. It is desirable to repeat the operations in detail for that panel point only at the upper extremity of the end posts (Fig. 11), and this point will be

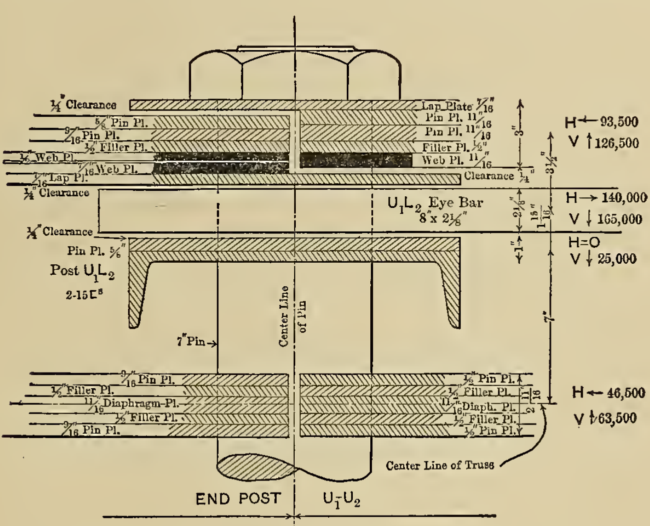


FIG. 11.

investigated for that position of loading which causes the maximum stress in U_1L_2; the following table shows the components of the direct stresses in the members meeting at U_1 for that condition of loading:

	Vertical Components.	Horizontal Components.
U_1L_1	+ 50,000 pounds	
L_0U_1	− 620,000 "	− 510,000 pounds
U_1U_2	+ 240,000 "	+ 790,000 "
U_1L_2	+ 330,000 "	− 280,000 "

No essential error will be introduced if it be assumed that there is no eccentricity in the line of action of L_0U_1 (the end post) and U_1U_2. The resulting differences between their stresses may therefore be taken exactly along their centre lines; a part of this stress is carried by the diaphragm; in this case about 34 per cent. Fig. 11 shows the stresses carried at one half of the panel point.

The resultant moments in inch-pounds are as follows:

Centre of Moments at Centre of	Horizontal Moment in Inch-pounds.	Vertical Moment in Inch-pounds.	Maximum Moment in Inch-pounds.
U_1L_2	← 306,000	↑ 415,000	515,000
U_1L_1	← 222,500	↑ 345,000	
Centre line of truss. .	→ 103,500	↓ 99,500	

The allowed bending resistance of a 7-inch pin is 606,100 inch-pounds; and this pin therefore satisfies the requirements at U_1.

Art. 23.—Camber.

§ 107. *All bridges shall be cambered by giving the panels of the top chord an excess of length in the proportion of $\frac{1}{8}$ inch to every 10 feet.*

In accordance with this specification the following table shows the original lengths and the changed lengths of the upper chord members. It is seen that the end post is not changed in length.

	Original Length.	Changed Length.
L_0U_1	46′ 4″	46′ 4″
U_1U_2	30′ $6\frac{9}{32}$″	30′ $6\frac{5}{8}$″
U_2U_3	"	"
U_3U_4	29′ $6\frac{1}{8}$″	29′ $6\frac{1}{2}$″
U_4U_5	"	"
U_5U_6	29′ 2″	29′ $2\frac{3}{8}$″

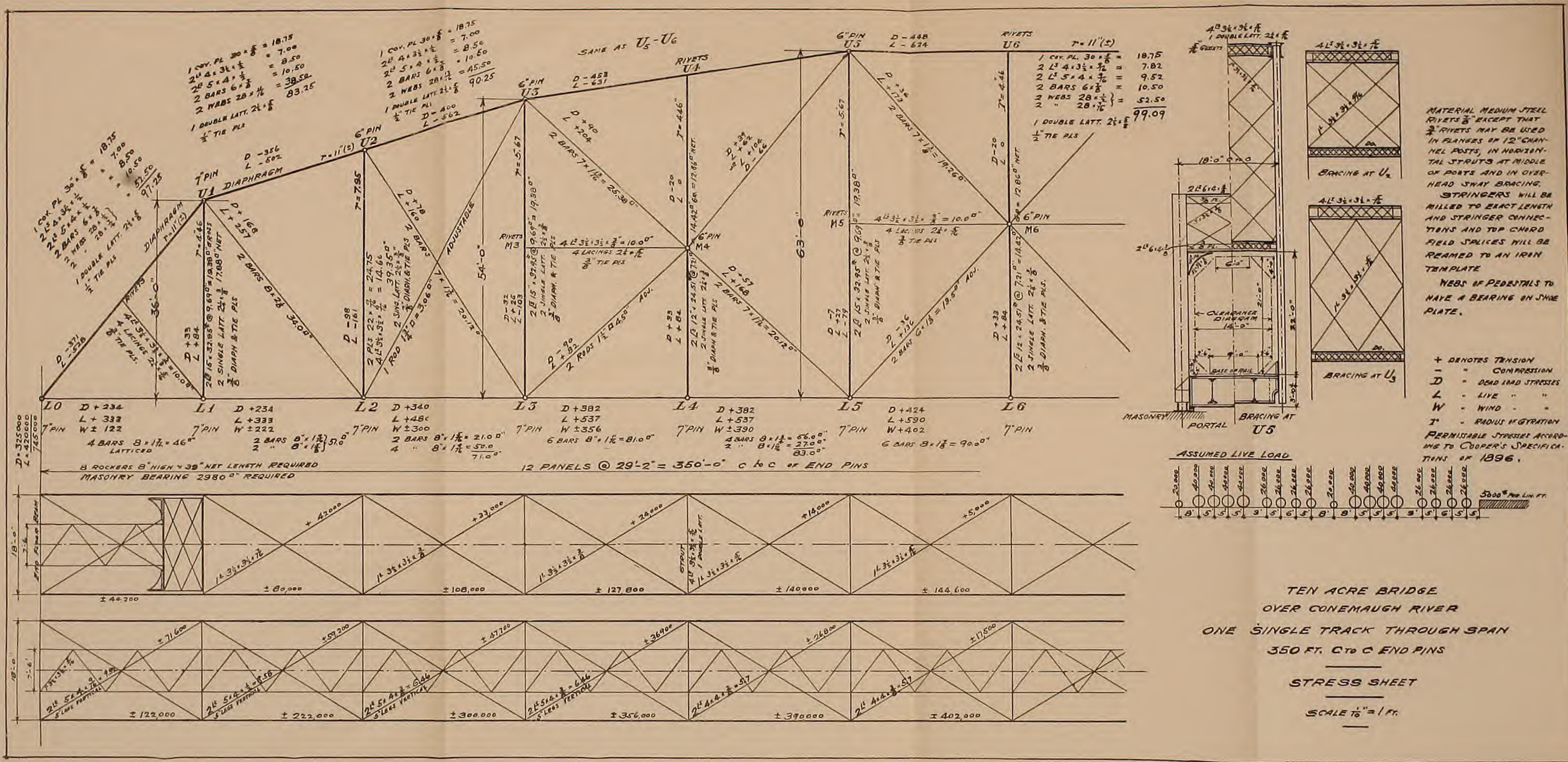
No changes are made in the lengths of the main web members, their final lengths being the same as if no changes had been made in the lengths of the upper chord members.

The stretching of the lower chord panels makes it necessary to increase the lengths of M_3M_4 and M_5M_6 to an amount equal to half that stretch. The final length of each of these members will then be 29 feet $2\frac{3}{16}$ inches.

Art. 24.—Conclusion.

The principles outlined indicate a system of rational treatment for the design of all parts of a modern truss bridge. It is obvious that a wide range of treatment of details is permissible in securing the above results, and those given here are subject to this general observation. Other details of equal efficiency might be designed but those given are at least as effective as others which might have been used and have served the purpose of illustration at least as well.

A carefully made bill of materials with the resulting computation of weights would show that the total shipping weight of this 350-foot single track through span would be 821,000 pounds.


TEN ACRE BRIDGE
OVER CONEMAUGH RIVER
ONE SINGLE TRACK THROUGH SPAN
350 FT. C TO C END PINS
STRESS SHEET
SCALE 1/16" = 1 FT.
12 PANELS @ 29'-2" = 350'-0" C to C OF END PINS
8 ROCKERS 8" HIGH × 39" NET LENGTH REQUIRED
MASONRY BEARING 2980 □" REQUIRED
ASSUMED LIVE LOAD
MATERIAL MEDIUM STEEL
RIVETS 7/8" EXCEPT THAT 3/4" RIVETS MAY BE USED IN FLANGES OF 12" CHANNEL POSTS, IN HORIZONTAL STRUTS AT MIDDLE OF POSTS AND IN OVERHEAD SWAY BRACING.
STRINGERS WILL BE MILLED TO EXACT LENGTH AND STRINGER CONNECTIONS AND TOP CHORD FIELD SPLICES WILL BE REAMED TO AN IRON TEMPLATE.
WEBS OF PEDESTALS TO HAVE A BEARING ON SHOE PLATE.
+ DENOTES TENSION
− " COMPRESSION
D - DEAD LOAD STRESSES
L - LIVE " "
W - WIND " "
r - RADIUS OF GYRATION
PERMISSABLE STRESSES ACCORDING TO COOPER'S SPECIFICATIONS OF 1896.
BRACING AT U2
BRACING AT U3
PORTAL
BRACING AT U5
MASONRY
CLEARANCE DIAGRAM
BASE OF RAIL
SAME AS U5-U6
DIAPHRAGM
ADJUSTABLE
L0
L1
L2
L3
L4
L5
L6
U1
U2
U3
U4
U5
U6
M3
M4
M5
M6
7" PIN
6" PIN
RIVETS
36'-0"
54'-0"
63'-0"
18'-0"


TEN ACRE BRIDGE
OVER CONEMAUGH RIVER
ONE SINGLE TRACK THROUGH SPAN
350 FT. C to C END PINS
GENERAL DETAILS
SCALE 3/4" = 1 FT.
12 PANELS @ 29'-2" = 350'-0" C-C END PINS
CENTER OF SPAN
SWAY FRAME AT U5

INDEX.

A.

B.

C.

I.

J.

K.

L.

M.

U.

V

W.